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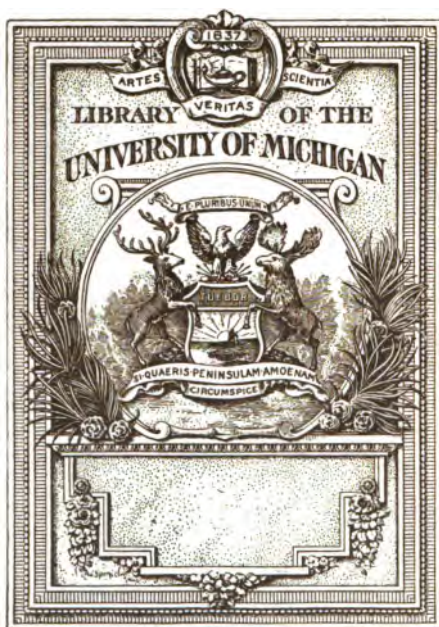
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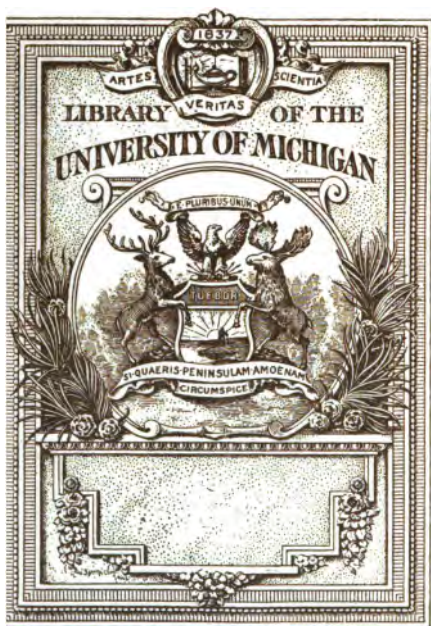
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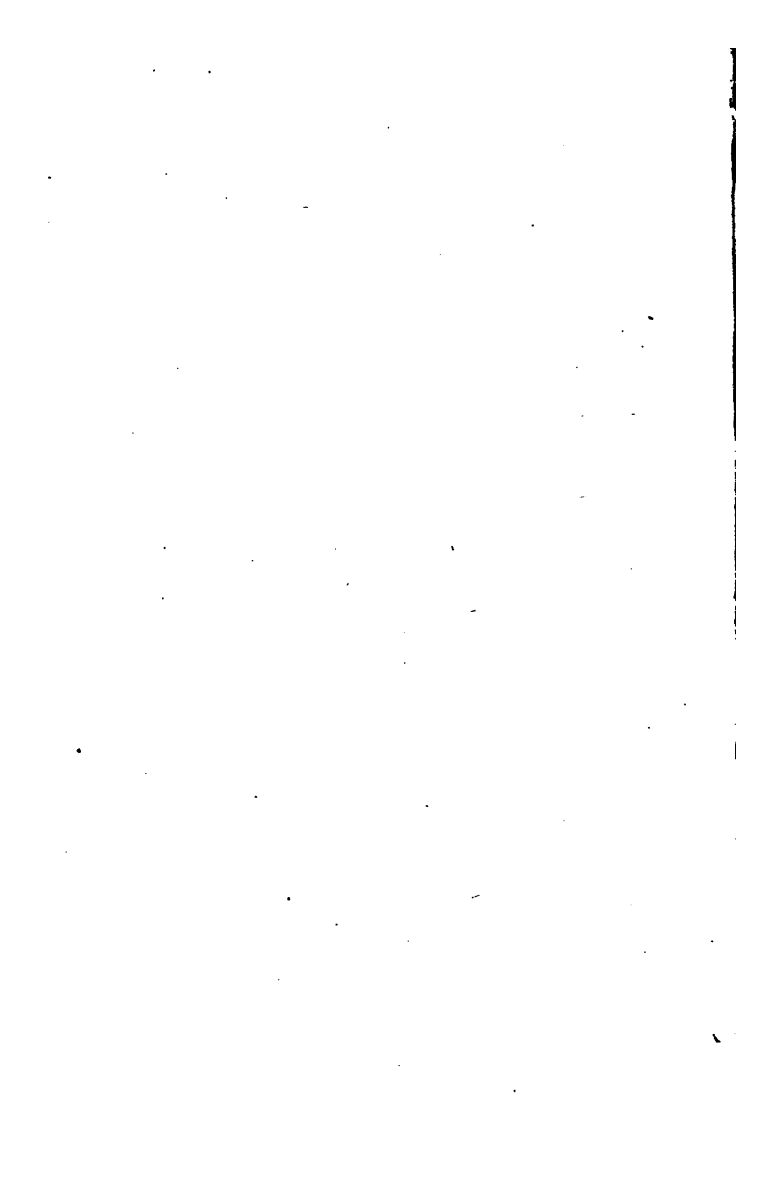
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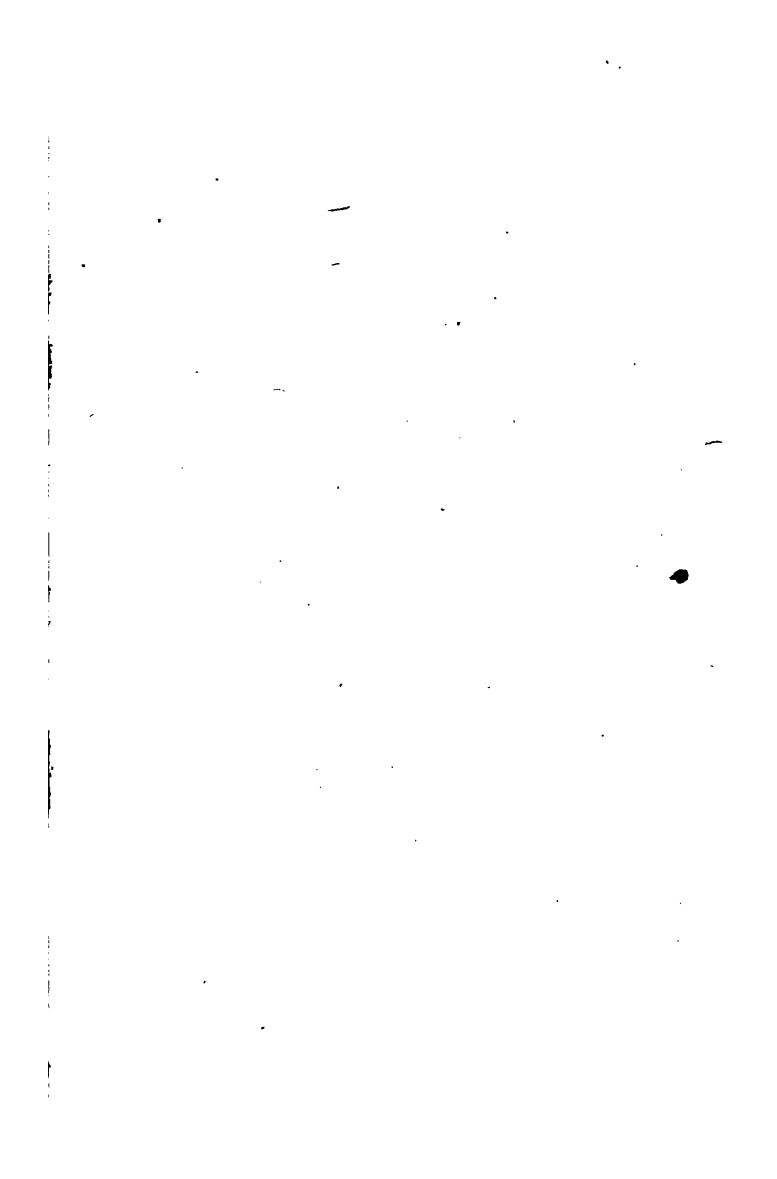


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BEAMS ^{AND} GIRDERS.

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PRACTICAL FORMULAS

FOR THEIR

RESISTANCE.

BY

Practical survey
P. H. PHILBRICK, C. E.,

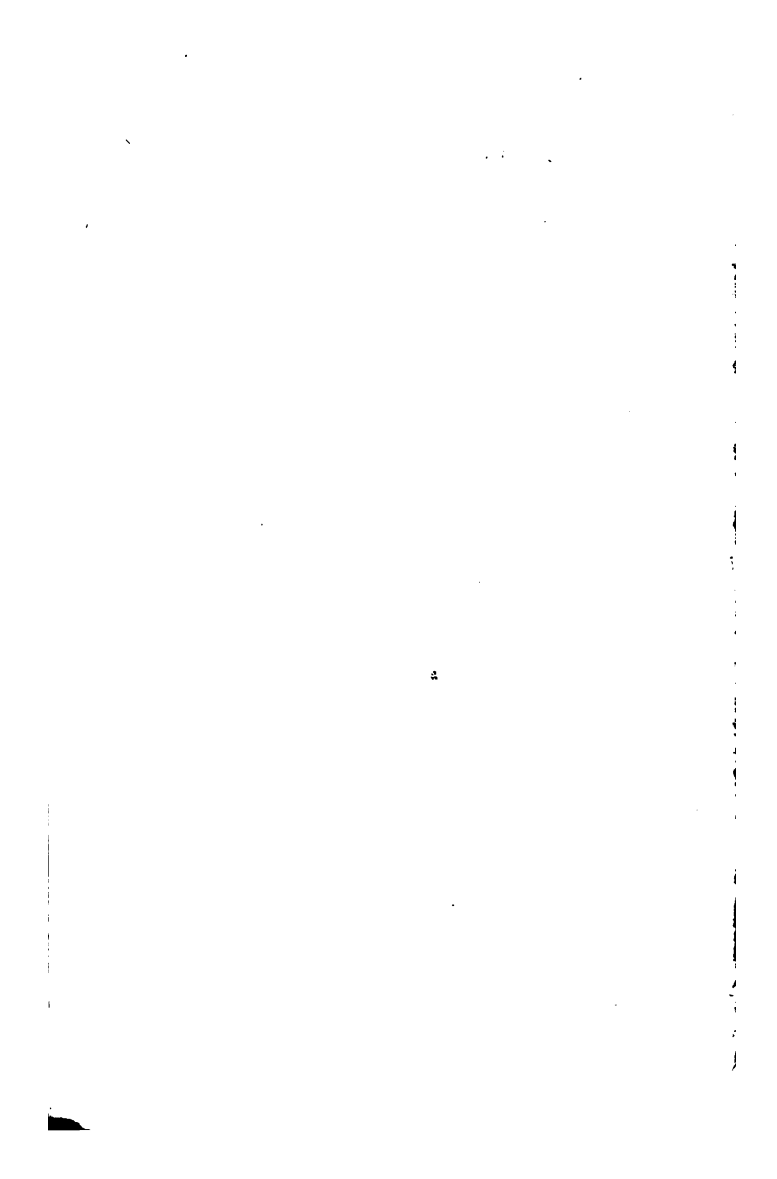
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PREFACE.

Revised... The main object of this treatise is to deduce general formulas for the resistance of beams and girders applicable to all cases, and to set forth truly "practical formulas," so far as seemingly required, in the use of existing forms and sections. These formulas are shorter and simpler than the corresponding ones in general use, and are, besides, practically perfectly accurate, which is by no means the case, as we have abundantly shown, with the common formulas.

The author is here led to remark that no truly practical formulas on any subject were ever produced except by those having a knowledge of the subject. Nor can there be a truly practical man—one who does things in his calling or profession in the best way—whose practice is not founded upon a personal knowledge

of the principles of that calling or profession.

Practice not founded on knowledge is guesswork, and is always awkward, round-about, and very expensive to those who pay for the guessing. Formulas devised by such "practitioners" are needlessly lengthy and incorrect, and well calculated to consume time and increase expense.

The principal errors in this subject are: 1st. Those arising from the use of erroneous formulas. These amount to 10 or 15%, or even much more in some cases. 2d. Those arising from erroneous estimates of the load coming upon beams, girders, etc. This is usually $12\frac{1}{2}\%$, but may be much less, as demonstrated in the text. The loads for given engines and wheel spaces are no doubt sometimes computed. But even then, unless those wheel spaces are such as to produce a maximum load on the beams, which is very improbable, the beams for a different spacing would be liable to a greater load, even up to the maximum, and would therefore be deficient in capacity, though not $12\frac{1}{2}\%$.

The combined error is usually not less than 25%, and *on the side of danger*.

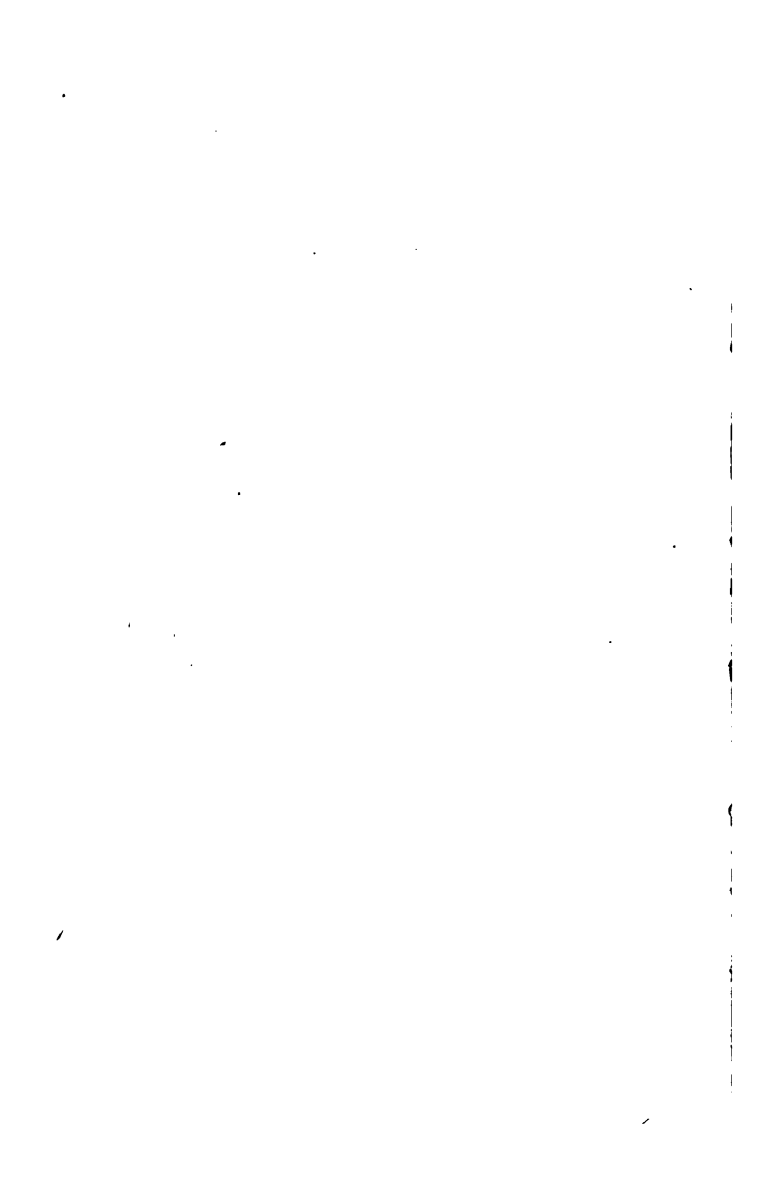
Deficiencies due to errors in sizes, or to imperfect workmanship, or to a poor quality of material, do not come within the province of this book.

Numerous applications of the formulas and comparisons with the usual ones are found in the book. The problems in designing are intended as approximate models, and also incidentally to illustrate in a general way the use of the formulas.

Appropriate tables are given for the proportioning of rivets, pins, joists, floor beams, stringers, etc.

The author submits the book to the judgment of the profession, hoping it may be a convenience, and that it may aid in correcting present errors and in avoiding others.

P. H. PHILBRICK.



BEAMS AND GIRDERS :

Practical Formulas for their Resistance.

By practical formulas is meant, those which are both simple and accurate, those which can be easily and safely used.

It is well known that the moment of inertia is a factor in most formulas involving the moment of resistance ; and that we usually arrive at the latter through the former. Otherwise we have little interest in the moment of inertia. The moment of resistance, however, does express something fundamentally essential; and the best formulas for the same are therefore very desirable. This will be clear as we proceed.

Accurate formulas for the moment of resistance of all usual sections are well known, but, being more or less complex, and therefore unmanageable by many Bridge Engineers and Architects, other

formulas more simple and *seemingly* quite correct, but unfortunately very erroneous, have come into general use.

The main objects of this treatise are :

1°. To deduce and set forth simple and accurate formulas for the resistance of beams, girders, etc.

2°. To point out, to some extent, the errors of the formulas in general use, and the reasons for the existence of these errors.

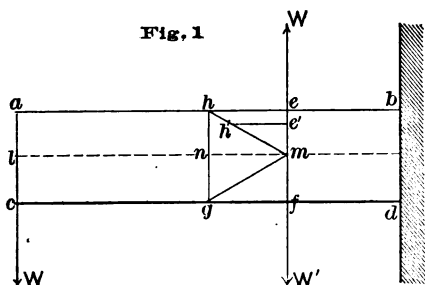
3°. To apply the new formulas in the examination of existing cases and in designing floor beams, track stringers, etc.

The nature and importance of the moment of resistance will be best understood by first considering the forces that, in ordinary cases at least, produce it. We may then advantageously confine the discussion to the moment of resistance itself.

I. Let $abcd$ (Fig. 1) represent a beam fixed at the end bd and loaded with a weight W at the free end ac . Length of beam $= ab = l$.

It is evident that the fibers on the

lower part of the beam will be compressed, and those on the upper part extended; and that, as a consequence, the beam will bend.



The fibers on some intermediate plane, lm , will therefore be neither compressed nor extended. This plane is called the neutral plane or axis.

The main points of the common theory of the resistance of beams to external forces in general are:

1°. The extensions and compressions of the fibers are directly as the forces that produce them.

2°. The forces, and therefore the com-

pressions and extensions also, are directly as their distance from the neutral axis.

3°. The neutral axis passes through the center of gravity of the section.

We are now prepared to explain the nature of the equilibrium between the external and internal forces.

Consider any section, ef , perpendicular to the axis of the beam.

Let b = the breadth, d = the depth and $x = lm$, the distance from the free end to any section considered.

We will first consider the external forces. Take moments about m . The moment of W is Wx , and this can be balanced only by the moments of the strains upon the fibers both above and below m , about the same point m . The force W also produces a shearing or cross strain, equal to W , upon every vertical section of the beam; and this strain can be met only by an equal stress upon the fibers in the same direction. By applying the equal and opposite forces W and W' on the section ef , the external strains may be made more apparent to some.

Since these forces balance each other, they cannot disturb, or in any way change the existing state of things. These forces, however, with the force W at the end of the beam, constitute the couple WxW , whose moment is $W \times lm = Wx$; and the force W' , which creates the shearing upon the section ef and upon all sections of the beam.

We see that the shearing strain in a beam loaded as in Fig. 1, is constant on all sections, but that the moment of strain Wx increases with x , or the distance of the section considered from the load. Hence at the fixed end the moment is a maximum, the fibers there are most strained, and the section at the fixed end is the "dangerous" section.

At the fixed end we have, the maximum moment $= M_0 = Wl$. (1)

Having found the moment of external forces upon a beam fixed at one end and loaded with a weight W at the free end, we will proceed to find an expression for the moment of internal forces.

Let s = the strain upon a unit of fibers

at e . Take $eh=s$. Were all the fibers on the cross section em of the upper half of the beam equally strained it is plain that the volume of the parallelopiped, whose face is $ehnm$ and depth is b (the breadth of the beam), would represent the tension upon the upper half of the beam. Since, however, the strains decrease regularly from the outermost fiber to the center, where the strain is zero, it follows that the strain on the upper half of the beam will be represented by the wedge whose face is ehm and breadth is b .

The resistance to tension is therefore equal to $\frac{1}{2} em \times eh \times b = \frac{1}{4} bds$.

The moment of this resistance is equal to this value multiplied by the distance of the center of gravity of the wedge from the center of the beam, which is equal to $\frac{2}{3}em = \frac{1}{3}d$. Hence the moment of resistance is equal to $\frac{1}{4}bds \times \frac{1}{3}d = \frac{1}{12}bd^2s$.

Since the moment of resistance to compression is the same, the total moment of resistance is therefore $M = \frac{1}{6}bd^2s$. (2)

Again, the moment of resistance of the

cross section em about its lower edge at m is, as just found, $\frac{1}{2} b d^2 s = \frac{1}{6} b (\frac{1}{2} d)^2 s$ (3).

This shows that the moment of resistance of a rectangle about one edge is equal to one-third ($\frac{1}{3}$) the breadth multiplied by the square of the depth and by the modulus of strain.

In the case of a beam, fixed at one end and loaded at the free end, we have seen that the moment of strain on the cross section at the fixed end of the beam is Wl ; and that the moment of the resisting forces on the fibers is $\frac{1}{6} b d^2 s$, the notation being as heretofore. These quantities must be equal, and hence

$$Wl = \frac{1}{6} b d^2 s \quad (4)$$

In all cases, the moment of external forces called the "bending moment" must be equal to the moment of internal forces, that is to the "moment of resistance."

s of course increases with W , and whether in any given case, s is as great as the material will stand, in which case it is at the point of rupture, depends upon the load W , that the beam has to

sustain, and upon the area and form of the cross section of the beam.

For a *given* load W , we solve for s and find,

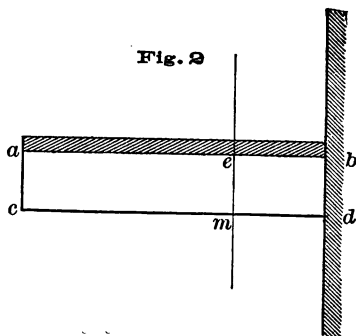
$$S = \frac{Wl}{\frac{1}{8}bd^3} = \frac{6Wl}{bd^3} \quad (5)$$

For a given or allowable value of s we solve for W and find

$$W = \frac{\frac{1}{8}bd^3s}{l} = \frac{bd^3s}{6l} \quad (6)$$

CASE II.

Let the beam be fixed at one end and uniformly loaded.



Let w = load per unit of length.

W = total load. Other notation as before.

Consider a section at em .

Load on $ae = wx$. The center of gravity of the load is at a distance from e equal to $\frac{1}{2}x$. Hence the moment of the load about e is equal to

$$M = wx \times \frac{1}{2}x = \frac{1}{2}wx^2.$$

At the fixed end

$$x = l \text{ and } M_0 = wl \times \frac{1}{2}l = \frac{1}{2}Wl \quad (7)$$

CASE III.

Let the beam (Fig. 3) be supported at the ends and carry a weight W at the middle point n . Notation as before. One half of the load will be carried to each support. Hence the reactions V and V_1 are each equal to $\frac{1}{2}W$.

The moment of strain on any section ef is $\frac{1}{2}Wx$. The greatest strain on the left hand half of the beam corresponds to the greatest value x can have for that half, and which is equal to $\frac{1}{2}l$. Hence

the greatest strain on that half, and therefore on the beam is,

$$M_0 = \frac{1}{2}W \times \frac{1}{2}l = \frac{1}{4}Wl \quad (8)$$

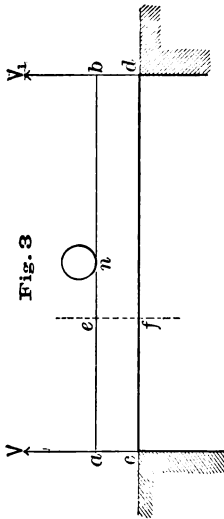


Fig. 3

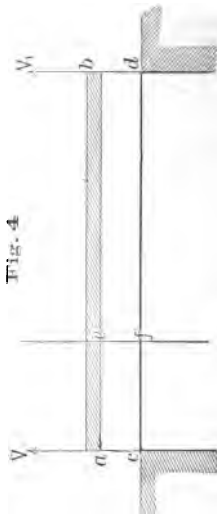


Fig. 4

CASE IV.

Let the beam (Fig. 4) be supported at the ends, and uniformly loaded as in Case II. Notation as usual. We evidently

have $V = V_1 = \frac{1}{2}W$. Hence at any section ef we have the moment of $V = \frac{1}{2}Wx$. The load between V and e is equal to $w x$, and its lever arm is $\frac{1}{2}x$; hence the moment of that load is $\frac{1}{2}w x^2$.

Since this moment acts in the opposite direction from the moment of V , their difference gives the moment on the section. Hence $M = \frac{1}{2}Wx - \frac{1}{2}w x^2$. Putting $x = \frac{1}{2}l$, this becomes

$$M_0 = \frac{1}{4}Wl - \frac{1}{8}wl^2 = \frac{1}{8}Wl \quad (9)$$

To prove that equation (9) gives the greatest strain upon the left hand half of the beam, and consequently upon the beam, since the beam is symmetrical about the center, put $x = \frac{1}{2}l - a$ in the value for M and we have:

$$M = \frac{1}{2}W(\frac{1}{2}l - a) - \frac{1}{2}w(\frac{1}{2}l - a)^2 = \frac{1}{4}Wl - \frac{1}{2}Wa - \frac{1}{8}wl^2 + \frac{1}{2}wla - \frac{1}{2}wa^2.$$

$$\text{But } \frac{1}{2}wla = \frac{1}{2}Wa \therefore M = \frac{1}{4}Wl - \frac{1}{8}wl^2 - \frac{1}{2}wa^2 = \frac{1}{8}Wl - \frac{1}{2}wa^2,$$

which is less than M_0 given by (9).

Substituting $\frac{1}{2}l \pm a$ for x will give precisely the same result and show that the strains are symmetrical with reference to

the center, which, in the nature of the case, we know to be true, and that the strain is a maximum at the center.

Putting the differential coefficient of $M = \frac{1}{2}Wx - \frac{1}{2}wx^2$ equal to zero, we have,

$$\frac{dM}{dx} = \frac{1}{2}W - wx = 0 \quad \therefore x = \frac{\frac{1}{2}W}{w} = \frac{1}{2}l,$$

for the point of maximum strain as already found.

Comparing equations (1), (7), (8) and (9), we see that, for the same total load, in the four cases considered, the moments of strain upon the sections of greatest strain, and consequently the moments of stress upon the fibers, as well as the stresses upon the fibers, are as the numbers 8, 4 2 and 1. Hence the beams are capable of carrying loads, disposed as shown, in the ratio of 1, 2, 4 and 8.

For example, a beam supported at the ends and uniformly loaded will carry eight times as great a load as the same beam will carry when fixed at one end and loaded at the free end.

In the first case, the shearing stress,

which we will represent by S , is constant and equal to $S_1 = W$ (10).

In the second case, the shearing stress at any section ef is evidently equal to the load between the free end and that section, and is $S = wx$. This is a maximum at the fixed end and equal to

$$S_1 = wl = W \quad (11)$$

In the third case, since $V = \frac{1}{2}W$ and there is no load between V and the load at the middle, the shearing stress at all points of the beam is evidently equal to

$$S_1 = \frac{1}{2}W \quad (12)$$

In the fourth case the reaction $V = \frac{1}{2}W$ and as the load between the free end and the section ef is equal to wx , the shearing stress on that section is equal to $S = \frac{1}{2}W - wx$.

At the end this is a maximum and

$$S_1 = \frac{1}{2}W \quad (13)$$

At the middle $S = \frac{1}{2}W - w\frac{1}{2}l = 0$.

Example 1.—Suppose the safe stress per square inch to be, $s = 1200$ pounds, $l = 16$ feet, $b = 3$ inches, and $d = 12$ inches.

What weight, uniformly distributed, will the beam, supported at its ends, carry?

From equations (2) and (9) we have:

$$\frac{1}{8}Wl = \frac{1}{8}bd^3s \text{ or } \frac{W = 4bd^3s}{3l}$$

$$= \frac{4 \times 3 \times 12^3 \times 1200}{3 \times 16 \times 12} = 3600 \text{ pounds.}$$

Example 2. — There are 9 joists $3'' \times 12'' \times 17'$ equally spaced, carrying 80 pounds per square foot on the floor of a bridge, which is 16 feet wide. What is the greatest strain per $\square''$ of fibers?

Each of the two outer joists carries but one-half as much as each of the others. Hence each of the middle joists carries $\frac{1}{8}$ of the total load or

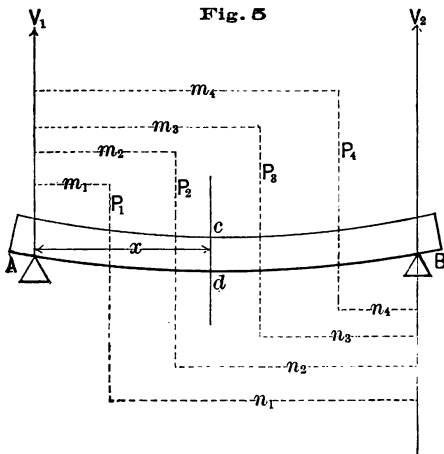
$$\frac{80 \times 16 \times 17}{8} = 2720$$

$$\text{Now } 2720 = \frac{4bd^3s}{3l} \text{ and } s = 963 \text{ pounds.}$$

OTHER CASES OF LOADING.

CASE V.—A beam resting upon two supports and loaded at any number of

points. If a beam is loaded in any manner it is only necessary to find the moments of all the forces in reference to the center of the section considered, and place the algebraic sum equal to the moment of resistance of the section. In Fig. (5), let the weights P_1, P_2, P_3 , etc.



rest upon the beam at distances respectively equal to m_1, m_2, m_3 , etc. from one

support; and n_1, n_2, n_3 , etc. from the other. Let l = the distance between the supports, and V_1 and V_2 the reactions at the supports. Consider any section, cd , distant x from the support A. The sum of the moments of the forces on the section cd is

$$V_1 \times -P_1(x-m_1) - P_2(x-m_2) - \text{etc.}$$

to include all the P 's between A and cd .

For rectangular beams we have therefore

$$V_1 x - P_1(x-m_1) - P_2(x-m_2) - \text{etc.} = \frac{1}{8} b d^3 s. \quad (14)$$

To find V_1 take moments about B and find,

$$V_1 l = P_1 n_1 + P_2 n_2 + P_3 n_3 + \text{etc.} = \Sigma P n$$

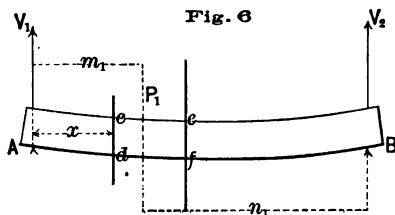
$$\therefore V_1 = \frac{\Sigma P n}{l}; \text{ similarly } V_2 = \frac{\Sigma P m}{l}.$$

$$\text{Also } V_1 + V_2 = P_1 + P_2 + P_3 + \text{etc.} = \Sigma P.$$

In case of a single weight P_1 , Fig. (6), the preceding formulas give,

$$V_1 = \frac{P_1 n_1}{l}, \quad V_2 = \frac{P_1 m_1}{l} \text{ and}$$

$$V_1 + V_2 = \frac{P_1}{l} (n_1 + m_1) = P_1.$$



From these we have for any section distant x from A, the moment equal to

$$M = V_1 x = \frac{P_1}{l} n_1 x. \quad (15)$$

Similarly for a point distant x from B,

$$M = V_2 x = \frac{P_1}{l} m_1 x. \quad (16)$$

Since $\frac{P_1}{l}$ is constant we see that the strain on *any* section varies as the product of the end segments, into which the beam is divided by the weight and the section.

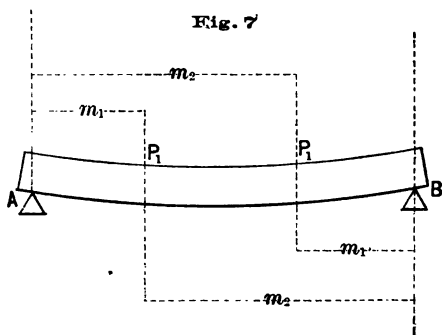
Again, making x the same in (15) and (16), we see that sections on opposite sides of the weight and equally distant from the supports, are strained in the ratio of the

segments into which the weight divides the beam.

In case of two equal weights P_1 and P_1 , symmetrically placed upon the beam as shown.

$$\text{We have } V_1 = \frac{P_1 m_2 + P_1 m_1}{l} = P_1.$$

$$\text{Similarly } V_2 = P_1.$$



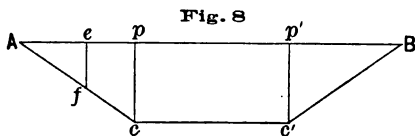
For any section between A and P_1 the moment is $M' = V_1 x = P_1 x$, which is less than $P_1 m_1$.

For any section between the weights, the moment $= V_1 x - P_1 (x - m_1)$

$$= P_1 x - P_1 (x - m_1) = P_1 m_1, \quad (17)$$

The strain therefore increases from zero at either support to the adjacent weight, and remains constant between the weights.

If, in Fig. 8, pc or $p'c'$ represents the



moment of strain under the weights, any ordinate as ef will represent the strain at the point e .

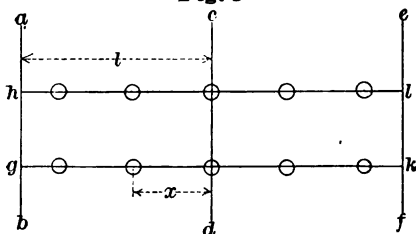
For a uniformly distributed load of $2P_1$, eq. (9) gives,

$$M'' = \frac{1}{8}l(2P_1) = \frac{1}{4}P_1l \quad (18)$$

Comparing eqs. (17) and (18), we see that the greatest strain in the case of a uniformly distributed load is to that of the same load when placed as in Fig. 7, as $\frac{1}{4}l$ is to m_1 .

Since tables are usually arranged to give the strains for a uniformly distributed load, on floor beams, track stringers, etc., it becomes important to notice

Fig. 9



and provide for increased strains due to concentrated loads as above.

Example.—A floor beam of a railway bridge is 14 feet long between supports and carries a single track, the gauge being 4 feet and 8 inches. The beam is designed to carry a distributed load just equal in amount to the eccentric load upon it. What is the capacity of the beam?

We have $m_1 = \frac{14 - 4\frac{2}{3}}{2} = 4\frac{2}{3}$. The load

on the beam is, $\frac{m_1}{\frac{1}{4}l} = \frac{4m_1}{l} = \frac{18\frac{2}{3}}{14} = \frac{3}{4}$ times the load it is able to carry. Hence it is able to carry $1 \div \frac{3}{4} = \frac{4}{3}$ of the given load; or it has a capacity of 75%.

It is worthy of notice that the greatest load that can come upon a floor beam, for example, for a given weight of engine per foot, depends upon the relative lengths of panel and wheel space, that is, the distance between consecutive pairs of wheels.

We will show that, in general, a load greater than the weight of the engine for a panel length, can come upon a floor beam; and we will deduce a general expression for the load and the excess of load over a panel load.

Let l = a panel length.

x = wheel space.

w = weight of engine per foot.

$\therefore wx$ = weight on a pair of wheels.

and wl = weight of engine per panel length.

Let $l > nx$ and $l < (n+1)x$, n being an integer.

In Fig. 9, let ab , cd , ef , etc., represent the floor beams, gk and hl the track stringers, and the circles the position of the wheels. Consider the load on cd .

Let a pair of wheels rest on cd . To find the load on cd we have wx resting directly upon cd , while $\frac{l-x}{l} wx$ comes upon cd from each of the adjacent pairs of wheels, distant x from cd ; or $2wx \frac{l-x}{l}$ from both pair. Similarly $2wx \frac{l-2x}{l}$ is supported by cd from the next two pairs of wheels, distant $2x$ from cd , etc., etc.

Hence the total load on cd is,

$$L = wx + 2wx \frac{l-x}{l} + 2wx \frac{l-2x}{l} + 2wx \frac{l-3x}{l} + \text{etc.} \dots \dots 2wx \frac{l-nx}{l}$$

or $L = wx + 2wx(1 + 1 + 1 + 1 + \text{etc. to } n$

terms) $= \frac{2n}{l}(x^2 + x^2 + 3x^2 + \text{etc. to } nx^2)$

$$= wx(2n + 1) - \frac{w}{l} n(n + 1)x^2 \quad (19)$$

Putting the differential coefficient of this equal to zero we find

$$(2n+1) - \frac{2n(n+1)}{l}x = 0$$

$$\text{or } x = \frac{(2n+1)l}{2n(n+1)} \quad (20)$$

Substituting this value of x in (19) and reducing, we find the maximum load on

$$cd = L_0 = wl \frac{(2n+1)^2}{4n(n+1)} = wl \frac{4n(n+1)+1}{4n(n+1)}$$

$$= wl + \frac{wl}{4n(n+1)} \quad (21)$$

L_0 is therefore greater than wl ; but the excess over wl decreases as n increases. For $n=0$ we have $L_0 = \frac{wl = \infty}{0}$.

But $n=0$ gives from above $x = \frac{l}{0} = \infty$ and

$\therefore wx = \frac{wl}{0} = \infty$ as it ought.

$$\text{From (20), } l - nx = \frac{l}{2(n+1)} \quad (22)$$

This shows the space between the n th pair of wheels from cd and the adjacent floor beam.

For $n=1$ (20) and (22) give, $x = \frac{3}{4}l$,
and $l - x = \frac{1}{4}l$.

For $n=2$ (20) and (22) give, $x=\frac{1}{2}l$,
and $l-2x=\frac{1}{2}l$.

For $n=3$ (20) and (22) give, $x=\frac{1}{3}l$,
and $l-3x=\frac{1}{3}l$.

For $n=1$ we have $L_0=\frac{2}{3}wl$.

For $n=2$ we have $L_0=\frac{2}{3}\frac{1}{2}wl$.

For $n=3$ we have $L_0=\frac{2}{3}\frac{1}{3}wl$.

For $n=4$ we have $L_0=\frac{2}{3}\frac{1}{4}wl$, etc.

These results show that the excess of load is slight, except for short panels and wide wheel spaces, in which case it may amount to $\frac{1}{3}$ or $12\frac{1}{2}\%$ of a panel load.

Example.—In a railway bridge, the length of panels is $l=10$ feet; the distance between pairs of wheels is $x=7\frac{1}{2}$ feet. Load, 4,000 pounds per foot. Find the greatest load supported by a floor beam and the excess over a panel load.

The load supported is

$$\frac{2}{3}wl = \frac{2}{3} \times 4,000 \times 10 = 45,000 \text{ pounds.}$$

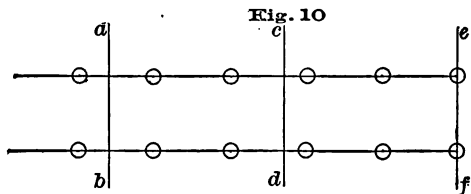
A panel load is

$$wl = 4,000 \times 10 = 40,000 \text{ pounds.}$$

The excess is therefore _____

$$\frac{1}{3}wl = 500 \times 10 = 5,000 \text{ po}$$

To prove that the maximum strain occurs when a pair of wheels rests upon the beam as in Fig. 9, let us suppose the engine to move to the right (say) a distance $= \frac{l}{2(n+1)}$ [see Eq. (22)], so that it will occupy the position shown in Fig. 10.



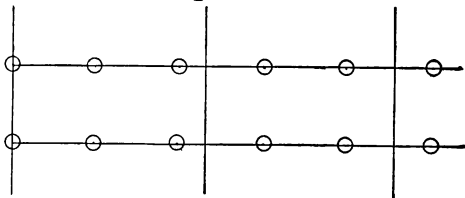
It is evident that only a part of the weight, wx , resting upon cd in Fig. 9 rests upon cd in Fig. 10, and that so far as the other weights are concerned, the increased load upon cd from the weights at the left of cd is exactly balanced by the decreased load upon cd from the weights at the right of cd . The load upon cd is less than in Fig. 9, by

$$\frac{\frac{l}{2(n+1)}}{l} wx = \frac{(2n+1)l}{2n(n+1)2(n+1)} w$$

$$= \frac{2n+1}{4n(n+1)} w \quad (23)$$

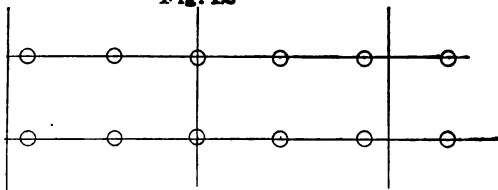
2°. Again let us suppose the engine to move from the position shown in Fig. 10 to that shown in Fig. 11. From what has been said, it is evident that no change in the load upon cd has taken place.

Fig. 11



3°. Finally we will suppose the engine to move from the position of Fig. 11 to that of Fig. 12. It is evident that this movement is the reverse of the first, and that the load will increase therefore during this change, precisely as it decreased during the first.

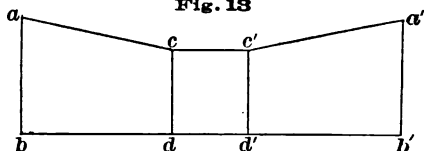
Fig. 12



In Fig. 13, let bb' represent a wheel space, then the ordinate ab represents the maximum load on the beam corresponding to Fig. 9, while cd and $c'd'$ represent the minimum loads corresponding to Figs. 10 and 11, and $a'b'$ the maximum corresponding to Fig. 12, which is the same as Fig. 9.

We see that while the engine advances a wheel space, the load upon a beam decreases for a part of that distance, remains constant during the second part, and increases again during the third part. The smallest loads upon the floor beam (cd), corresponding to the positions shown in Figs. 10 and 11, or any position between these two, are not minimum loads, but simply the loads correspond-

Fig. 13



ing to certain positions of the engine, when the wheels are spaced to produce a maximum load.

To find the wheel space for a minimum load upon a floor beam, when a pair of wheels is over a beam. Eq. (20) gives the value of x for a maximum load.

Put $x = \frac{2n+1}{2n(n+1)}l + a$ in (19) and we find

$$\begin{aligned} L' &= \frac{(2n+1)^2}{2n(n+1)}wl + wa(2n+1) \\ &\quad - \frac{w}{l}n(n+1)\left(\frac{2n+1}{2n(n+1)}l + a\right)^2 \\ &= \frac{(2n+1)^2}{4n(n+1)}wl - \frac{w}{l}n(n+1)a^2 \quad (24) \end{aligned}$$

Eqs (21) and (24) give,

$$L_0 - L' = \frac{w}{l}n(n+1)a^2 \quad (25)$$

This shows that the greater a is, the less the load upon the beam.

$$\text{Now } l = nx + (l - nx) = n \left(x + \frac{l - nx}{n} \right)$$

This shows that if the wheel space is increased by the n th part of the excess of a panel length over n wheel spaces, n times the increased wheel space will equal a panel length.

$$\text{Hence } a \text{ cannot exceed } \frac{l - nx}{n}$$

But Eq. (22) gives,

$$\frac{l - nx}{n} = \frac{l}{2n(n+1)} = a \quad (26)$$

Putting this value of a in (24) we get,

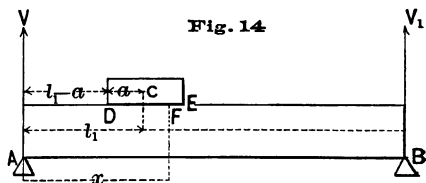
$$\begin{aligned} L' &= \frac{(2n+1)^2}{4n(n+1)} wl - \frac{w}{l} n(n+1) \frac{l^2}{4n^2(n+1)^2} \\ &= \frac{(2n+1)^2}{4n(n+1)} wl - \frac{1}{4n(n+1)} wl = wl \quad (27) \end{aligned}$$

The minimum load upon the beam corresponds to wheel spaces equal to a panel length, or to an aliquot part of the same; the minimum load being equal to a panel load.

It is not so necessary to consider the increased strains upon floor beams, etc., in case of a double track, though we may just as easily do so; for, in the first place, the loads upon the beams are much more nearly distributed than in the case of a single track; and, furthermore, the loads on the two tracks would rarely occupy the same position with reference to the floor beam.

A PARTIAL UNIFORM LOAD.

Let a beam be uniformly loaded over *any* portion of its length.



Let $l = AB$, the length of the beam,
 $2z = DE$, the length of the load,
 $x = AF$, the distance to any section,
 C = the center of the load,
 $l_1 = AC$, w = load on a unit of length,
 V = the reaction of support A.

Then $AD = l_1 - a$ and $DF = x - (l_1 - a)$
 $= x + a - l_1.$

The load on $DF = w(x + a - l_1).$

The load on $DE = 2wa.$

By the principle of moments,

$$Vl = 2wa(l - l_1) \quad \therefore V = 2wa \left(1 - \frac{l_1}{l}\right)$$

The moment of stress at F is,

$$M = Vx - w(x + a - l_1) \frac{1}{2}(x + a - l_1)$$

$$= Vx - \frac{1}{2}w(x + a - l_1)^2,$$

$$\text{or } M = 2aw \left(1 - \frac{l_1}{l}\right) x - \frac{1}{2}w(x + a - l_1)^2 \quad (28)$$

That value of x which makes (28) a maximum gives the position of the dangerous section.

Differentiate, place equal to zero, and solve for x and find,

$$x = a \left(1 - \frac{2l_1}{l}\right) + l_1 = l_1 \left(1 - \frac{2a}{l}\right) + a$$

$$= a + l_1 - \frac{2al_1}{l} \quad (29)$$

$$\text{If } l_1 = \frac{1}{2}l, \quad x = l_1.$$

$$\text{If } l_1 < \frac{1}{2}l, \quad x > l_1.$$

$$\text{If } l_1 > \frac{1}{2}l, \quad x < l_1.$$

These results show that the maximum strain is at the center of the loading only when the center of the loading is over the center of the beam, and that in all other cases it is nearer the center of the beam than the center of the loading is.

To find the relative positions of the centers of loading and dangerous section, put $l_1 = \frac{1}{2}l \pm d$; then

$$x = a \left(1 - \frac{l \pm 2d}{l} \right) + \frac{1}{2}l \pm d = \pm d \frac{l-2a}{l} \pm \frac{1}{2}l.$$

$$\text{Now } x - \frac{1}{2}l = \pm d \frac{l-2a}{l} \text{ and } l_1 - \frac{1}{2}l = \pm d.$$

$$\text{Hence } \frac{x - \frac{1}{2}l}{l_1 - \frac{1}{2}l} = \frac{l-2a}{l} \quad (30)$$

which shows the ratio of the distances of the dangerous section and the center of loading from the center of the beam.

Differentiating (29) with respect to l_1 and also with respect to a , we find,

$$\frac{dx}{dl_1} = 1 - \frac{2a}{l} \quad (31)$$

which is positive; and

$$\frac{dx}{da} = 1 - \frac{2l_1}{l} \quad (32)$$

which is positive for $l_1 < \frac{1}{2}l$ and negative for $l_1 > \frac{1}{2}l$.

Eq. (31) shows that x increases and decreases with l_1 .

Eq. (32) shows that when $l_1 < \frac{1}{2}l$, that is, when the center of loading is between the center of the beam and the origin, x increases and decreases with a ; but when $l_1 > \frac{1}{2}l$, that is, when the center of loading is on the opposite side of the center of the beam from the origin, x increases as a decreases and vice versa.

The maximum strain for given values of a and l_1 is found by substituting the value of x from (29) in (28). We have

$$x = a + l_1 - \frac{2al_1}{l} \quad \therefore x + a - l_1 = 2a\left(1 - \frac{l_1}{l}\right)$$

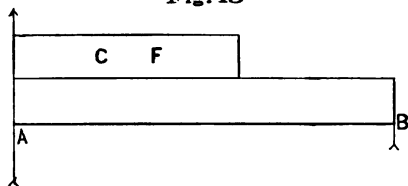
$$\begin{aligned} \text{Hence } M_0 &= 2aw\left(1 - \frac{l_1}{l}\right)\left(a + l_1 - \frac{2al_1}{l}\right) \\ &\quad - 2a^2w\left(1 - \frac{l_1}{l}\right)^2 \\ &= 2aw\left(1 - \frac{l_1}{l}\right)\left\{a + l_1 - \frac{2al_1}{l} - a\left(1 - \frac{l_1}{l}\right)\right\} \\ &= 2aw\left(1 - \frac{l_1}{l}\right)\left(l_1 - \frac{al_1}{l}\right) \\ &= 2awl_1\left(1 - \frac{l_1}{l}\right)\left(1 - \frac{a}{l}\right) \quad (33) \end{aligned}$$

The value of a which will make (33) a maximum is found by differentiating the equation with reference to a . We thus find $1 - \frac{2a}{l} = 0$ or $2a = l$.

Hence the larger the load the greater the strain.

When $2a = l$, $l_1 = \frac{1}{2}l$ and $M = \frac{1}{2}wl^2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}wl^2 = \frac{1}{8}Wl$, agreeing with Eq (9) as it ought.

Fig. 15



Let $a = \frac{1}{4}l$ and $l_1 = \frac{1}{4}l$

Then the strain at F is by (33),

$$M_0 = \frac{1}{2}wl \cdot \frac{1}{4}l \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{128}wl^2.$$

To find the strain at the center, we have the load $= \frac{1}{2}wl$; and $V = \frac{3}{4} \cdot \frac{1}{2}wl = \frac{3}{8}wl$.

Taking moments about the center of the beam we have $M' = \frac{3}{8}wl \times \frac{1}{2}l - \frac{1}{2}wl \times \frac{1}{4}l$
 $= \frac{8}{128}wl^2$

This shows that the strain at the dangerous section for this loading exceeds that at the center by $\frac{1}{8}$, or 12½%. This excess is the same as the maximum excess coming upon a floor beam from an undistributed load.

We will now derive a general formula for the moment of resistance, after which we will deduce the moment of resistance of triangular and other cross sections.

Let s = strain on a unit of fibers, of any cross section, most remote from the neutral axis.

d = distance from the neutral axis to the most remote fiber.

y = distance to any fiber.

δ = an elementary area.

Then $\frac{s}{d}$ = strain on a unit of fibers at a distance unity.

$\frac{s}{d}y$ = strain on a unit of fibers at a distance y .

$\frac{s}{d}\delta$ = strain on an element of fibers at a distance y .

$\left(\frac{s}{d}y\right)y$ = moment of strain on an element of fibers at a distance y .

This expression, taken for all values of y within the section, will give the moment of strain of the given section. Let M represent this moment. Then

$$M = \sum \left(\frac{s}{d}y \right) y = \sum \frac{s}{d} y^2 \delta = \frac{s}{d} \sum y^2 \delta \quad (34)$$

In (34) the factor $\sum y^2 \delta$ is called the moment of inertia of the cross section and is represented by I . It is obtained by multiplying each elementary area by the square of its distance from the neutral axis and taking the sum of the products. Representing $\sum y^2 \delta$ by I , we have,

$$M = \frac{s}{d} I \quad (35)$$

We must not, however, suppose that the moment of resistance involves the product of the strains on the different parts of the cross section into the *squares* of their distances from the neutral axis. The moment of resistance is obtained by multiplying each elementary strain $\left(\frac{s}{d}y\right)$

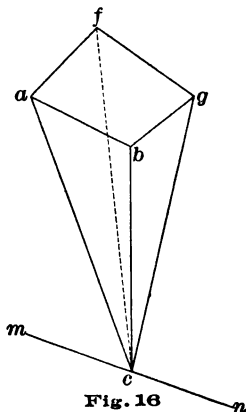
by its distance from the axis (y). The y in eq. (34) comes from the factor y in the factor of *strain* and the y which expresses the *distance* from the axis.

$$\text{When } s=1, \text{ we have } M_1 = \frac{I}{d} \quad (36)$$

The quantity $\frac{I}{d}$, that is, the moment of inertia of the surface, divided by the distance of the furthestmost fiber from the neutral axis, might with entire propriety be called the unit moment of resistance; for it does represent the moment of resistance of the section corresponding to a unit of strain upon a unit of area of fibers most remote from the neutral axis. Since, however, s varies with the nature and quality of the material, while $\frac{I}{d}$ depends only upon the area and form of section, the quantity represented by $\frac{I}{d}$ can be conveniently tabulated while that represented by $s\frac{I}{d}$ cannot so well be. The quantity $\frac{I}{d}$ is, therefore, frequently called

the moment of resistance, though $s \frac{I}{d}$ is the real moment of resistance.

The strain upon a unit of fibers most remote from the neutral axis and represented by s , is called the "modulus of strain," or "maximum fiber strain." We may frequently refer to either $\frac{I}{d}$ or $s \frac{I}{d}$ as the moment of resistance or simply resistance, since this will lead to no confusion.



To find the moment of resistance of the triangle abc about an axis mn passing through the vertex c and parallel to ab .

Let $ab=b$ and bc =the altitude of the triangle= d .

Draw af and hg , each equal to unity, perpendicular to the plane abc .

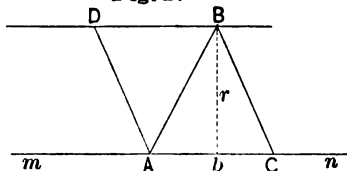
It is plain that the total strain upon the triangle is represented by the volume of the pyramid $abfg-c$, which is equal to $ab \times hg \times \frac{1}{3}bc = \frac{1}{3}bd$; and that the moment of resistance is represented by the static moment of the pyramid. The center of gravity of the pyramid is at a distance from mn equal to $\frac{3}{4}d$. Hence the moment of resistance is equal to

$$M = \frac{1}{3}bd \times \frac{3}{4}d = \frac{1}{4}bd^2 \quad (37)$$

To find the moment of resistance of a triangle ABC (Fig. 17) about an axis coinciding with the base; we observe that the moment of resistance of this triangle is equal to that of the parallelogram $ABCD$ less that of the triangle ABD . Hence, from eqs. (3) and (37)

$$M_1 = \frac{1}{3}bd^2 - \frac{1}{4}bd^2 = \frac{1}{12}bd^2 \quad (38)$$

Fig. 17



To find the resistance of a triangle having angles at 45° at A and C, and therefore 90° at B.

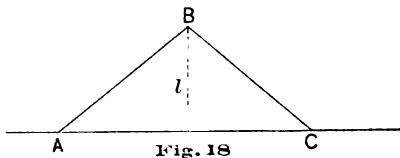


Fig. 18

We have $d = \frac{1}{2}b$ or $d^2 = \frac{1}{4}b^2$, and hence

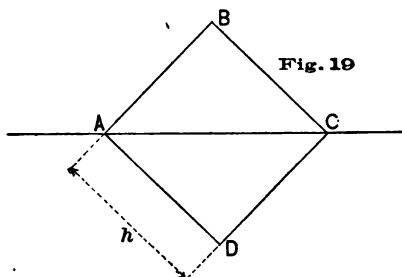
$$M_1 = \frac{1}{12}bd^2 = \frac{1}{48}b^3.$$

If $BC = h$, $b^2 = 2h^2$ and $b^3 = 2.828 h^3$,
and, therefore,

$$M_1 = \frac{1}{48} \cdot 2.828 h^3 = .059 h^3 = .06 h^3,$$

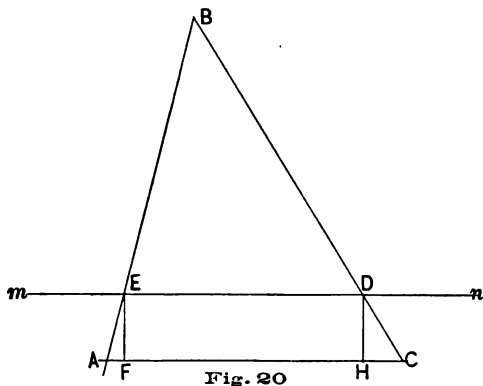
very nearly. (39)

To find the resistance of a square, whose side is h , about a diagonal. From (39), we have,



$$M_1 = .118 h^3 = .12 h^3, \text{ nearly.} \quad (40)$$

To find the moment of resistance of a triangle about an axis through the center



of gravity. Drop the perpendiculars EF and DH upon the base. Observe that $DE = \frac{2}{3}b$, $AF + CH = \frac{1}{3}b$; the altitude of $BED = \frac{2}{3}d$ and $EF = \frac{1}{3}d$. Also that the strain along the line AC is only half as much as at the point B. Hence the moment of resistance of

$$BED = \frac{1}{2} \left(\frac{2}{3}b \right) \left(\frac{2}{3}d \right)^2 = \frac{16}{24 \times 27} b d^3$$

the moment of resistance of

$$EAF \times DHC = \frac{1}{2} \text{ of } \frac{1}{4} \left(\frac{1}{3}b \right) \left(\frac{1}{3}d \right)^2 = \frac{3}{24 \times 27} b d^3$$

and the moment of resistance of

$$EDFH = \frac{1}{2} \text{ of } \left(\frac{2}{3}b \right) \left(\frac{1}{3}d \right)^2 = \frac{8}{24 \times 27} b d^3$$

Hence the moment of resistance of

$$ABC = M_1 = \frac{1}{24} b d^3 \dots \quad (41)$$

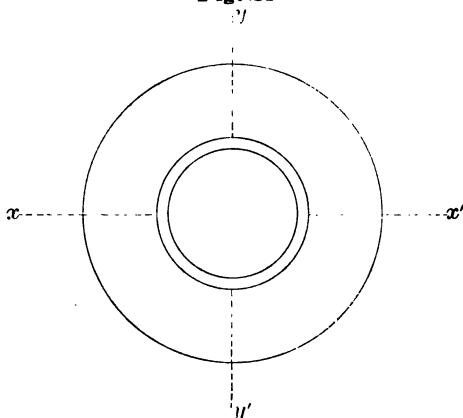
Since the equation $M_1 = \frac{I}{d}$ or $dM_1 = I$

is perfectly general, the moment of inertia is equal to the moment of resistance multiplied by the distance from the axis to the outermost fiber; or the moment of resistance is equal to the moment of inertia divided by the same quantity.

To find the polar and plane moments of inertia of a circle.

The polar moment is the moment about an axis through the center perpendicular to its plane.

Fig. 21



Let r = radius, y and $y + dy$ the radii of two smaller circles concentric with the other. The area of the annulus is, $\delta = 2\pi y dy$, and the square of its distance

from the center being y^2 , the polar moment of inertia of the circle is,

$$I_p = 2\pi \int_0^r y^3 dy = \frac{1}{2} \pi r^4 \quad (42)$$

Now, considering *any* elementary area, it is plain that its polar moment is equal to the sum of its moments about XX' and YY' .

The polar moment of the *entire* circle is, therefore, equal to the sum of its moments about XX' and YY' .

But these latter are evidently equal, and hence either is equal to half the polar moment.

$$\text{Hence} \quad I = \frac{1}{2} \pi r^4 \quad (43)$$

The moments of resistance are found, of course, by dividing these expressions by r .

$$\text{Hence} \quad M_p = \frac{1}{2} \pi r^3 \quad (44)$$

$$\text{and} \quad M_i = \frac{1}{4} \pi r^3 \quad (45)$$

To find the polar moment of resistance directly, we have $\delta = 2\pi y dy$ as above. The strain on a unit of area of the outermost fibers of the circle being unity, the strain on a unit of fibers at the distance y , is $\frac{y}{r}$. Hence the strain on the annu-

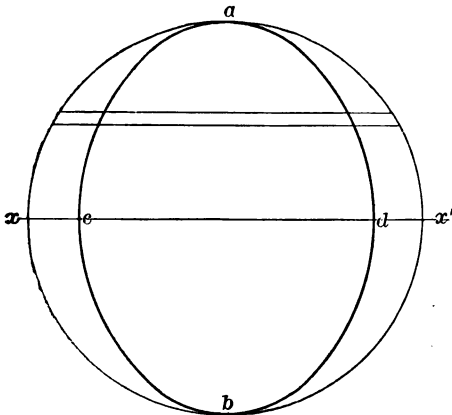
lus is, $2\pi y dy \frac{y}{r} = \frac{2\pi}{r} y^2 dy$; and the moment of that strain is $\frac{2\pi}{r} y^3 dy$. Hence

$$M_p = \frac{2\pi}{r} \int_0^r y^3 dy = \frac{2\pi}{r} \cdot \frac{r^4}{4} = \frac{1}{2} \pi r^3$$

and $M_i = \frac{1}{4} \pi r^3$.

To find the moment of resistance of an ellipse about its shorter axis, cd (Fig. 22).

Fig. 22



Let $ab = 2a$ and $cd = 2b$.

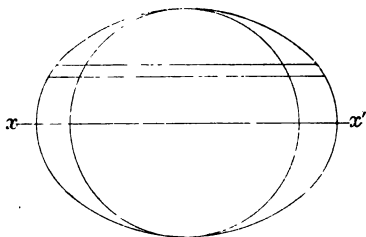
Circumscribe a circle about the ellipse.

The resistance of the circle is,

$\frac{1}{4}\pi r^3 = \frac{1}{4}\pi a^3$. But considering *any* elementary strip parallel to XX' , the length of that strip, included within the ellipse, is to the length included within the circle as b is to a . Hence the resistance of the ellipse is

$$M_1 = \frac{1}{4}\pi a^3 \frac{b}{a} = \frac{1}{4}\pi a^2 b \quad (46)$$

Fig. 23



For an ellipse about its longer axis inscribe a circle. The notation being as before we have for the circle

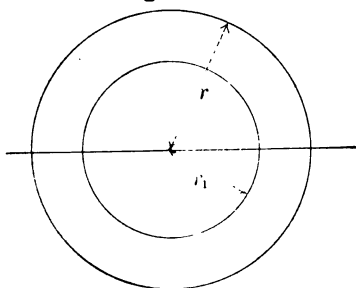
$$M_1 = \frac{1}{4}\pi b^3,$$

and, therefore, for the ellipse,

$$M_1 = \frac{1}{4}\pi b^3 \frac{a}{b} = \frac{1}{4}\pi b^2 a \quad (4)$$

A hollow circle (Fig. 24) or annulus.

Fig. 24



To find the moment of resistance of a section equal to the hollow, we observe that the resistance on the outer limit of the hollow is not unity, but only $\frac{r_1}{r}$.

Hence the moment of resistance by (45) is

$$\frac{1}{4}\pi r_1^3 \times \frac{r_1}{r} = \frac{1}{4}\pi r_1^4 \quad (48)$$

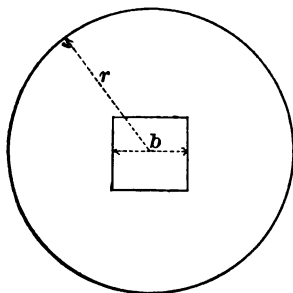
That is, the moment of resistance of the hollow is equal to its moment of inertia divided by the distance of the axis from the farthest fiber of the whole section. Hence by (45) and (48) the mo-

ment of resistance of the whole section is

$$M_1 = \frac{1}{4}\pi r^3 - \frac{\frac{1}{4}\pi r_1^4}{r} = \frac{\frac{1}{4}\pi r^4 - \frac{1}{4}\pi r_1^4}{r} \quad (49)$$

Hence the moment of resistance of a hollow section is equal to the difference of the moments of resistance of the whole section and that of the hollow; or it is equal to the difference of the moments of inertia of the same sections divided by the distance from the axis to the farthest fiber.

Fig. 25

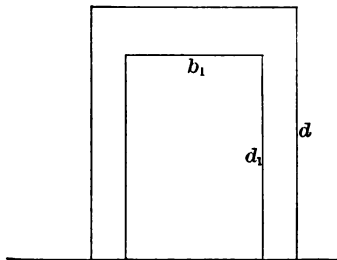


The moment of resistance of the section shown in Fig. 25, for example, is,

$$M_1 = \frac{1}{4}\pi r^3 - \frac{1}{8}\frac{b^4}{r} = \frac{\frac{1}{4}\pi r^4 - \frac{1}{8}b^4}{r} \quad (50)$$

Moment of resistance of a hollow rectangle (Fig. 26).

Fig. 26



Eq. (3) gives the moment of resistance of the whole section $= M' = \frac{1}{8}bd^3$.

The strain at b_1 is equal to $\frac{d_1}{d}$, therefore for the hollow

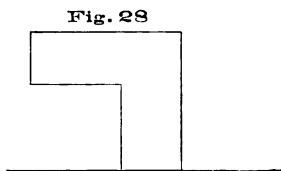
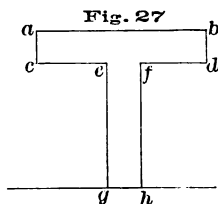
$$m'' = \frac{1}{8}\frac{d_1}{d}b_1d_1^2 = \frac{1}{8}\frac{b_1d_1^3}{d}.$$

Hence for the real section

$$m = m' - m'' = \frac{1}{8}\frac{bd^3 - b_1d_1^3}{d} \quad (51)$$

Formula (51) evidently applies to the

"T" section as shown in Fig. 27 or to the "angle" as shown in Fig. 28.



If $d = \frac{1}{2}b$ and $d_1 = \frac{1}{2}b_1$, (51) becomes

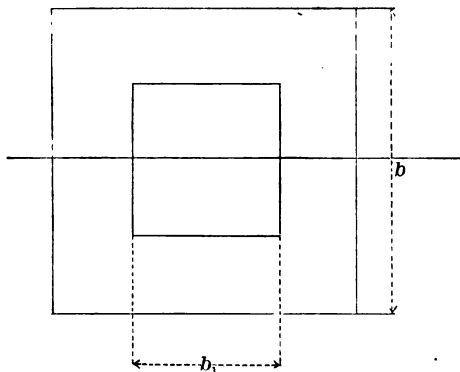
$$M_1 = \frac{1}{12} \frac{b^4 - b_1^4}{b}, \quad (52)$$

Hence for a hollow square Fig. 29,

$$M_1 = \frac{1}{6} \frac{b^4 - b_1^4}{b}. \quad (53)$$

If $b_1 = b$, the web vanishes and equation (51) gives for the flange alone,

Fig. 29



$$\begin{aligned}
 M_1 &= \frac{1}{8} \frac{b(d^3 - d_1^3)}{d} = b(d - d_1) \frac{d^2 + d_1 d + d_1^2}{3d} \\
 &= b(d - d_1) \frac{3dd_1 + (d - d_1)^2}{3d} \\
 &= b(d - d_1) \left[d_1 + \frac{(d - d_1)^2}{3d} \right]. \quad (54)
 \end{aligned}$$

Let $d - d_1 = h$ and $b(d - d_1) = a = \text{area of the flange.}$

$$\begin{aligned}
 \text{Then } M_1 &= a \left(d - h + \frac{1}{8} \frac{h^2}{d} \right) = a \left(d_1 + \frac{1}{8} \frac{h^2}{d} \right) \\
 &= a \left\{ \frac{(d - \frac{1}{2}h)^2}{d} + \frac{h^2}{12d} \right\} \quad (55)
 \end{aligned}$$

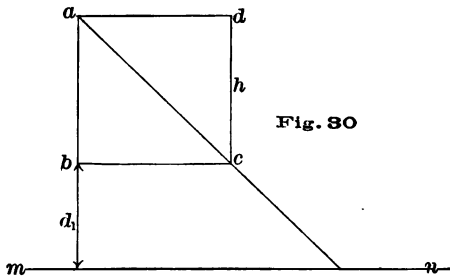
$$= a \frac{(d - \frac{1}{2}h)^2}{d} \text{ nearly.} \quad (56)$$

In a similar way we find for the triangle adc (Fig. 30), $M = a \left(d - \frac{2}{3}h + \frac{h^2}{6d} \right)$ (57)

and for the triangle abc

$$M = a \left(d - \frac{1}{3}h + \frac{h^2}{2d} \right) \quad (58)$$

in which $a =$ the area of the triangle.



From (55) we have the moment of inertia of a rectangle about an axis outside of the section, as shown in Fig. 27,

$$\begin{aligned} = I &= ad \left(d - h + \frac{h^2}{3d} \right) = a \left\{ (d - \frac{1}{2}h)^2 + \frac{h^2}{12} \right\} \\ &= a \left\{ g^2 + \frac{h^2}{12} \right\} = ar^2 \text{ say} \quad (59) \end{aligned}$$

in which $g = d_1 + \frac{1}{2}h$ = the distance from the neutral axis to the center of gravity of the section.

r is called the radius of gyration, and expresses the distance from the axis to a point, at which, could the whole area of the rectangle be placed, the moment of inertia would not be changed.

Dividing by a and extracting the square root of (59), to two terms, gives

$$v = g + \frac{h^2}{24g} \quad (60)$$

If the axis passes through the center, $g = 0$, and (55) gives

$$r = \frac{h}{\sqrt{12}} = .289 h \quad (61)$$

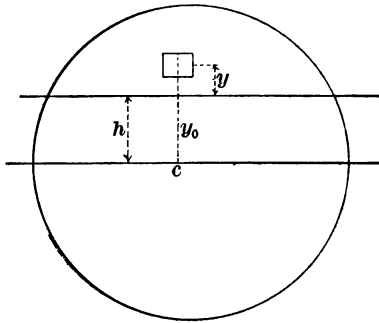
If the axis passes through one edge $g = \frac{h}{2}$ and

$$r = \frac{h}{\sqrt{3}} = .577 h \quad (62)$$

FORMULAS OF REDUCTION.

To find a general expression for the moments of inertia, I , and of resistance M_r , about any axis (Fig. 31).

Fig. 31



Let y = distance of any element from the axis.

y_0 = the distance of any element from the parallel axis through the center of gravity, and h the distance between the axes.

Let I_0 = the moment of inertia about the axis through the center of gravity. Let the elementary area be represented by dA , A being the area of the section.

Now $I_0 = \int y_0^2 dA$, and

$$I = \int y^2 dA = \int (y_0 - h)^2 dA = \int y_0^2 dA - \int 2hy_0 dA + \int h^2 dA = I_0 + h^2 A \quad (63)$$

Since $\int y_0 dA$, which expresses the sum of the products of all the elements of area into their distances from the axis, is equal to zero.

Again, let d_0 = the distance of the farthest element from the axis through the center of gravity, and d the similar distance from the parallel axis. Let M_0 and M , represent the moments of resistances about the axis through the center of gravity and the parallel axis.

Now $I = Md$ and $I_0 = M_0 d_0$.

Hence from (63)

$$Md = M_0 d_0 + Ah^2 \text{ or } M = \frac{M_0 d_0}{d} + \frac{Ah^2}{d} \quad (64)$$

(64) is the formula of reduction for the moments of resistances.

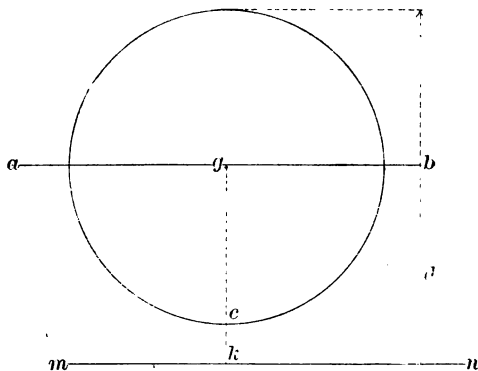
To find the moment of resistance of a circle about any axis mn (Fig. 32).

Let r = radius, a = the area of the circle and g , d_0 and d as before.

Let I_0 = the moment of inertia about ab and I = the moment of inertia about mn .

$$\text{Now } I_0 = \frac{1}{4} \pi r^4 = \frac{1}{4} a r^2.$$

Fig. 32



$$\begin{aligned}
 I &= I_0 + ag^2 = a[(d-r)^2 + \frac{r^2}{4}] \\
 &= a(d^2 - 2dr + \frac{5r^2}{4}) \\
 \therefore m &= \frac{I}{d} = a \left(d - 2r + \frac{5r^2}{4d} \right) = a \left(d_1 + \frac{5r^2}{4d} \right) \quad (65)
 \end{aligned}$$

Putting $2r = h$, (65) becomes

$$M = a \left(d - h + \frac{5h^2}{16d} \right) = a \left(d_1 + \frac{5h^2}{16d} \right) \quad (66)$$

To find the moment of resistance of an ellipse about any axis, parallel to a diameter of the ellipse.

Let $2b$ and $2r$ represent the axes, $r > b$, and suppose r perpendicular to the axis of moments. Then, in the same way as for the circle we deduce

$$\begin{aligned} m &= a \left(d - 2r + \frac{5}{4} \frac{r^2}{d} \right) = a \left(d_1 + \frac{5}{4} \frac{r^2}{d} \right) \\ &= a \left(d_1 + \frac{5}{16} \frac{h^2}{d} \right) \end{aligned} \quad (67)$$

This shows that the center of moments of an ellipse is the same as of a circle whose diameter coincides with the axis of the ellipse, which is perpendicular to the axis of moments.

A semi-circle (Fig. 33).

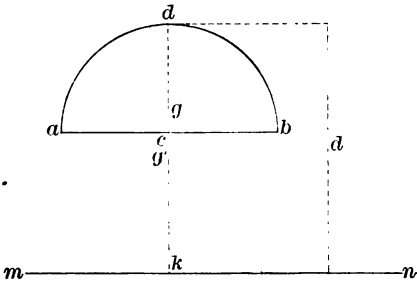


Fig. 33

Let r =radius, a =area, and let g be the center of gravity.

Let I , I_1 and I_0 represent the moments of inertia about mn , ab and an axis through g parallel to ab or mn .

From Mechanics we have $cg = .424r$

$$\text{Now } I_1 = \frac{\pi}{8} r^4$$

$$\begin{aligned} \therefore I_0 &= \frac{\pi}{8} r^4 - \frac{\pi}{2} r^2 \overline{cg}^2 = \frac{\pi}{8} r^4 - .09\pi r^4 \\ &= .035\pi r^4 = .11r^4 \text{ very nearly.} \end{aligned}$$

Hence

$$\begin{aligned} I &= .035\pi r^4 + \frac{\pi}{2} r^2 \times \overline{gk}^2 = .035\pi r^4 + \\ &\frac{\pi}{2} r^2 (d - .576r)^2 = .20\pi r^4 + \frac{\pi}{2} r^2 (d^2 - 1.15dr) \end{aligned}$$

$$\begin{aligned} \text{Therefore } M &= \frac{\pi}{2} r^2 \left(.40 \frac{r^2}{d} + d - 1.15r \right) \\ &= \frac{\pi}{2} r^2 \left(d - 1.15r + \frac{2r^2}{5d} \right) \quad (68) \end{aligned}$$

Putting $h = 2dg = dy = 1.15r$, this becomes,

$$m = a \left(d - h + \frac{3}{10} \frac{h^2}{d} \right) \quad (69)$$

A semicircle Fig. 34.

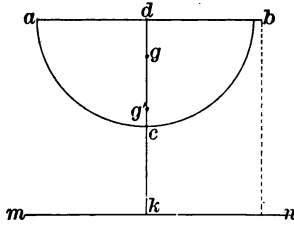


Fig. 34

We have $I_c = .035\pi r^4$

$$\text{and } I = .035\pi r^4 + \frac{\pi}{2} r^2 \bar{g} k^2$$

$$= .035\pi r^4 + \frac{\pi}{2} r^2 (d - .424r)^2$$

$$= \frac{\pi r^4}{8} + \frac{\pi}{2} r^2 (d^2 - .848dr)$$

$$\therefore m = \frac{\pi}{2} r^2 \left(d - .848r + \frac{r^2}{4d} \right) \quad (70)$$

Putting $h = 2dy = dy' = .848r$ we have

$$m = a \left(d - h + \frac{1.04h^2}{3d} \right) \quad (71)$$

Thus we see that for the same values of d and h , (or of d and g , since $y = d - \frac{1}{2}h$) the lever arms of all these sections are very nearly the same, since the terms

$$\frac{1}{8}\frac{h^2}{d}, \frac{1}{16}\frac{h^2}{d}, \frac{2}{8}\frac{h^2}{d} \text{ etc.}$$

are approximately the same

We observe, too, that since the terms are quite small, the lever arm is in all cases, approximately, as much less than the distance to center of gravity of the section as this distance is less than the distance to the farthest fiber of the section.

$$\begin{aligned} \text{From (55), } M_1 - ad_1 &= \frac{ah^2}{3d}, \text{ or } m - a\left(g - \frac{h}{2}\right) \\ &= a\frac{h^2}{3d} \text{ or } ag - m_1 = a\left(\frac{h}{2} - \frac{h^2}{3d}\right) \end{aligned}$$

But $m_1 - ad_1$ = the difference between ad_1 and the true moment m_1 , or the error of ad_1 and $ag - m_1$ shows the error of ag .

$$\text{Now } \frac{ag - m_1}{m_1 - ad_1} = \left(\frac{1}{2} - \frac{h}{3d}\right) \div \frac{h}{3d} = \frac{3d}{2h} - 1 \quad (72)$$

Since d is usually several times larger than h , (72) shows that ag differs from the true moment several times as much as ad_1 differs; yet ag is the value gener-

ally taken for m , instead of the equally simple and more accurate value ad .

It is plain that if each unit of area was strained equal to unity, the total strain would be equal to the area a , and the moment of strain would be equal to ag ; but the outermost fibers only are so strained, and as the strain decreases uniformly toward the axis, the total strain is consequently much less than a (though the lever arm is indeed a little greater than g) and the moment of strain, or resistance, is much less than ag , as we have shown.

We may suppose, however, the strain equal to the area, which is *greater* than the real strain, provided we use a lever arm correspondingly *less* than the real lever arm. This is what the equations do, and they may be so interpreted and understood. The lever arm, by which the area is multiplied, may be called the reduced lever arm.

The important and practical principle to be remembered is this: that the lever arm of the moment extends from the

axis to a point quite as far inside of the center of gravity as the center of gravity is inside of the outermost fiber.

We will now apply the preceding formulas to the different forms of beams, etc.

EYE BEAMS.

To find the moment of resistance about an axis mn , through the center of gravity of the section and parallel to ab . The

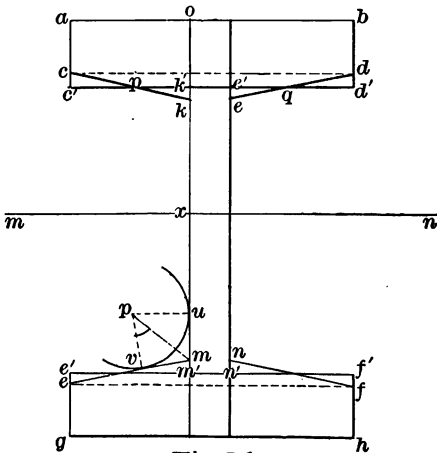


Fig. 35

flanges are usually trapezoidal. We will for the present consider the web as extending through the whole depth of the beam.

Let $abdlmfghgemkc$ represent the eye-beam. Through the middle points, p and q , of ck and ld , draw the line $c'k'l'd'$; also through the middle points of em and fn draw $e'm'n'f'$. Consider for the present $abd'l'n'f'hge'm'k'c'$ in place of the original figure.

Let $h' = ac$, the least thickness of the flanges.

$H = ok$, the greatest thickness of the flanges.

$t = kl$, the thickness of the web.

$b = ab - kl =$ width of flanges.

$h = \frac{H + h'}{2} =$ average thickness of the flanges.

Let $D = ag =$ total depth.

$d = ox =$ one half of total depth.

$D_1 = c'e' =$ inside depth of new figure or average depth of original figure.

$d_1 = k'x$, one half of the inside depth.

Let a = area of each flange $= b \frac{h' + H}{2}$
 $= bh$

B = [area of web $= t.D$]

Now, by number 2 of table 1, the moment of resistance of the web on either side of the axis is $\frac{1}{8} \times \frac{1}{2} B \times \frac{1}{2} D = \frac{1}{16} BD$; and by (55) the moment of resistance of

either flange $= a \left(\frac{1}{2} D_1 + \frac{h^2}{3 D_1} \right)$

Hence the total moment $= m = D_1 a + \frac{1}{8} DB + \frac{4ah^2}{3D}$ (73)

or since $a = b \frac{H + h'}{2}$ and $h = \frac{H + h'}{2}$

$$m = D_1 a + \frac{1}{8} DB + \frac{b}{6D} (H + h)^2 \quad (74)$$

But it is easy to show that the moment of resistance of the new figure exceeds that of the original figure by $\frac{bD_1}{6D} (H + h')^2$

Hence $m = D_1 a + \frac{1}{8} DB + \frac{b}{6D} [(H + h)^2 - D_1 (H - h)^2]$. (75)

$= D_1 a + \frac{1}{8} BD$ very nearly. (76)

Eq. (76) applies at once to built beams, but the flanges are not the same as for solid beams and hence the value of D_1 for built beams remains to be pointed out.

With reference to (75), we remark that the quantities within the parentheses are necessarily small, their difference is there-

fore necessarily small and the factor $\frac{b}{6\bar{D}}$

being but a small fraction, the product of this by the quantity within the brackets is, in practical cases, but a fraction of a unit—usually a very small fraction.

If in (76) we ignore the web we have,

$$m = D_1 a \text{ very nearly} \quad (77)$$

$$\text{or } m = D a \text{ approximately.} \quad (78)$$

Let us now suppose the web to terminate inside the flanges, and let A = the area of a flange, and b = the area of the web.

$$\begin{aligned} \text{Then we may write } m &= AD_1 + \frac{1}{3}bD_1 \\ &= D_1(A + \frac{1}{3}b). \end{aligned} \quad (79)$$

The term AD_1 is slightly too small for the flange and $\frac{1}{3}bD_1$, too large for the web.

On the whole the formula gives results very nearly correct, but slightly in excess.

If we ignore the web altogether, but use D instead of D_1 for the lever arm of A , we have $m = AD$. (80)

Equation (79) is the proper practical formula for solid beams in all cases, though (76) may be used when greater accuracy is desired, which can scarcely be the case, however. It is a significant fact that equation (80) which ignores the web, and requires but one multiplication, produces results *more* accurate than the *most* accurate formulas in general use. We will now set forth the usual formulas and then exhibit the merits and demerits of them a.l. in a general comparison.

If in (76) we substitute for both D and D_1 , the distance g between the centers of gravity of the flanges, we get,

$$m = g(a + \frac{1}{6}B) \quad (81)$$

This is used by J. A. L. Waddell and others.

Writing g for D_1 in 79 we get

$$m = g(A + \frac{1}{6}b) \quad (82)$$

Writing g for the lever arm of the flange in (79) we find,

$$m = Ag + \frac{1}{6}bD_1 \quad (83)$$

This is the same as equation (5), Article 65, of Prof. Burr's "Elasticity and Resistance of the Materials of Engineering." It is intended for use, and is extensively used. It is highly recommended by A. P. Boller, in his Highway Bridges, page 111.

If we consider the flanges for their whole depth, and the web as extending to the *middle* of the flanges and represent it by b' , we may write,

$$m = g(A + \frac{1}{6}b'). \quad (84)$$

Omitting allowances for extra diagonal resistances, recognised by Mr. Daniel K. Clark and others, the above is Mr. Clark's formula for eye beams, etc.

To show how erroneous the common formulas are, we will give the true resistances of the 13 eye beams rolled by Carnegie, also the resistances by our equations (76), (79), (80). By the side of these, we will give the resistances by

equations (81) and (82) and add an example to illustrate equations (83) and (84).

MOMENT OF RESISTANCE.

Number of Beam.	True Moment.	Moment by (76).	Moment by (79).	Moment by (80).	Moment by (81).	Moment by (82).
1	70.60	70 50	72 05	66.00	72.96	76.81
2	90.38	90.23	92.70	85.80	93.66	100.09
3	45.96	45 95	47.09	44.50	47.68	50.62
4	31.80	31.41	32 11	29.79	32.55	34.28
5	29.97	29.96	30.51	31.03	31.65	33.07
6	21.68	21.68	22.41	22.59	22.90	23.84
7	35.34	35.23	35.40	36.13	37.08	40.41
8	17.50	17.47	17 81	18.10	18.46	19.37
9	13.13	13.08	13.55	13.35	14.00	14 52
10	8.16	8 16	8.31	8.50	8 62	9.02
11	4.94	4.93	5.03	5.12	5 21	5.47
12	3.09	3.08	3.15	3.25	3 27	3.46
13	2.06	2.03	2.07	2.50	2.27	2.39

Omitting No. 13, which is not a practical beam, we observe that our equation (76) gives results almost exact; and that (79) is also practically accurate; while equations (81) and (82) give results in excess of about 5% and 10% respectively, and are therefore too erroneous to be

countenanced in good practice. Besides, they involve far more labor than the far more accurate formulas recommended above. Thus are they doubly condemned and the use of them therefore doubly absurd.

The following from Boller's Highway Bridges, page 111, will illustrate equations (83) and (84).

$$t = \frac{1}{2}, D_1 = 13, h = 1, b = 5, D = 15$$

$$\therefore A = 5, b = 6.5 \text{ and } g = 14$$

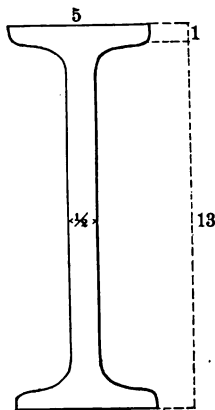


Fig. 36

Hence by (83)

$$m = 14 \times 5 + \frac{13 \times 6.5}{6} = 70 + 14.083 \\ = 84.083$$

Now the true value of M is

$$\frac{5 \times 15^3 - 4.5 \times 13^3}{6 \times 15} = 77.65$$

Hence the error $= 6.43$

Now $6.43 \div 77.65 = 8.3\%$ nearly

By equation (84), we have

$$A = 5, b' = 14 \times \frac{1}{2} = 7 \text{ and } g = 14.$$

Hence, $m = 14(5 + \frac{1}{3}) = 86.33$

Therefore this is $86.33 - 77.65 = 8.68$
in error.

This is an error of $8.68 \div 77.65 = 11.2\%$
nearly.

The formula deduced by Mr. Clark is,
 $m = g(A + .28888b')$ or $m = 14(5 + 2.0216)$
 $= 98.30$

The error in the former result shows, however, the total error of the result due to mathematical errors in the corresponding formula.

We observe that equation (80) does indeed give results more accurate than any

of the common formulas in use. This shows that a direct though random shot may rest nearer the center than others obliquely though deliberately aimed. Since the errors of these common formulas far exceed the allowable limit, it follows that the errors involved in their use are not fully appreciated by those who use them.

For example, the author of a desirable treatise on "Designing of Ordinary Iron Highway Bridges" uses Eq. (83) and considers it safe; though, comparing it with equation (76), we see that the excess given by it for the flanges exceeds many times the deficiency given for the web.

The author from whom Eq. (83) and the example under it are quoted, remarks that: "the above process for obtaining the value of R [that is, the moment of resistance, which we have represented by m] varies so fractionally from absolute truth, that the refinement of calculation, necessary to obtain mathematical exactness, is entirely unnecessary."

Eq. (83) gives, however, only a rough

approximation to the truth, while Eq. (76) shows that a *proper* equation with no "refinement of calculation" whatever, is all that is necessary to obtain a practically perfect result. Indeed (76) is of the same form as (83), though (76) does not, for reasons already given, involve g . It is believed that all the practical formulas, so called, heretofore used contain the standard errors on this subject.

So far, we have said nothing about the metal in the curved corners, which it is customary and proper to take no account of. We will, however, show how the moment of resistance of the corners may be easily obtained.

In Fig. (35), draw the radii uP and vP and join Pm . Let the angle $mPv = \theta$.

$$\begin{aligned} \text{The area of } vmuP &= um \times uP = (uP)^2 \\ &\tan \theta = r^2 \tan \theta. \end{aligned}$$

$$\begin{aligned} \text{The area of the sector } vuP &= \frac{1}{2} \text{ arc } vu \\ &\times uP = (uP)^2 \theta = r^2 \theta. \end{aligned}$$

Therefore the area of the curved corner $= r^2 (\tan \theta - \theta)$, and the area of the four corners $= 4r^2 (\tan \theta - \theta)$.

Also $mx = \frac{1}{2}D - H$, and the strain at m is equal to $\frac{\frac{1}{2}D - H}{\frac{1}{2}D} = \frac{D - 2H}{D}$.

Hence the moment of resistance of the four curved areas is, very nearly,

$$4r^2 (\tan \theta - \theta) \frac{D - 2H}{D} (\frac{1}{2}D - H) \\ = 2r^2 (\tan \theta - \theta) \frac{(D - 2H)^2}{D}$$

This averages about 2% in Carnegie's beams Nos. 1 and 2 and less than 1% in the others.

The moment of resistance of eye beams about any axis is best found by finding the moment of resistance of the flanges and the web (or parts of the flanges or web, if the axis passes through flange or web) separately, and adding; or, if the moment m about mn is known we may use the formula of reduction (64).

CHANNEL BARS.

The preceding formulas apply directly to channel bars, a or A being the area of a flange, or total area of flanges on one

side of the axis, when two (or more) channels are used; B or b the area of a web or of the webs.

Since, however, the flanges of channels are thin, compared with the depths of the channels, those formulas give results for channels even more accurately than for beams.

CENTER OF GRAVITY.

Since the neutral axis is assumed to pass through the center of gravity of the section, it becomes necessary to find, in some cases, the position of the center of gravity, or at least of the axis passing through it, in order to find the moments of resistance and of inertia.

The center of gravity of a body may be defined as the center of the forces of gravity, assumed to be parallel, acting upon the body.

If the center of gravity of a body is supported, the body will be supported, for the forces of gravity balance about the center of gravity.

For our purpose, it will be necessary

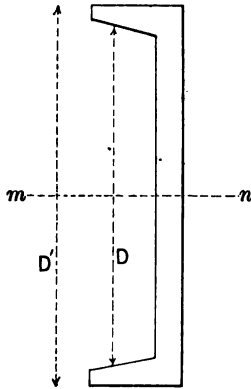


Fig. 37

to consider, to a limited extent only, the center of gravity of areas. The center of gravity of any regular polygon is at the center of the inscribed or circumscribed circle of a parallelogram at the intersection of the diagonals; of a plane triangle on the line joining the vertex with the middle of the base and at one-third the length of this line from the base; of a circle or ellipse, at the geometrical center of the figure, etc.

We observe that the center of gravity of any section symmetrical with reference to an axis, will be found on that axis; so that for such sections it is only necessary to find one line upon which the center of gravity is situated, the center of gravity being at the intersection of this line with the *symmetrical* axis.

To find the center of gravity of a double "T" eye beam with unequal flanges.

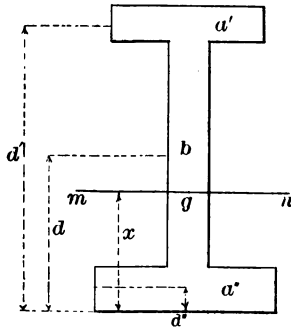


Fig. 38

Let a' be the area of the upper flange, a'' that of the lower flange and b that of

the web, as shown in Fig. 38, d' , d'' and d , the distances of their centers of gravity from the *base* of the beam, and x the same for the whole figure.

Let A =the total area. Then taking moments about the base we have:

$$(a' + a'' + b)x = bd + a'd' + a''d''$$

$$\text{or } x = \frac{bd + a'd' + a''d''}{a' + a'' + b} \quad (85)$$

The center of gravity is, of course, on the center line of the web.

Or, let the lever arms be measured from the center of gravity of the lower flange as indicated in Fig. 39.

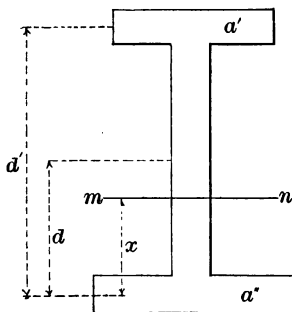


Fig. 39

In this case $d' = 0$, and we have :

$$x = \frac{bd + a'd'}{a' + a'' + b} \quad (86)$$

If h = the thickness of the lower flange, the distance from the base of the beam to its center of gravity is,

$$x = \frac{1}{2}h + \frac{bd + a'd'}{a' + a'' + b} \quad (87)$$

These equations apply equally to the case in which the web is neglected, by putting $b = 0$ in them.

Considering the web extending the whole depth of the beam, as in Fig. 40,

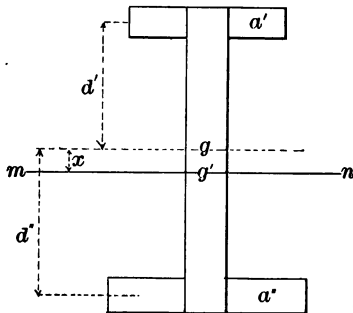


Fig. 40

we may easily find the distance of the center of gravity from the center of the beam.

The areas being represented as before, let g be the center of gravity of the web, and g' that of the beam. Let $gg' = x$ and let d' and d'' be the distances from the centers of the flanges to the center of the web.

Taking moments about g , we have :

$$\begin{aligned}
 (a' + a'' + b)x &= a''d'' - a'd' \\
 \text{or } x &= \frac{a''d'' - a'd'}{a' + a'' + b} \quad (88)
 \end{aligned}$$

The above formulas apply without change to Deck Beams.

Let Fig. 41 represent a single "T."

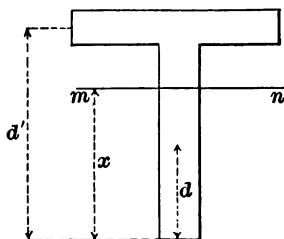


Fig. 41

Take moments about the end of the stem and find the distance from the end of the stem to the center of gravity of the section, $x = \frac{bd + a'd'}{a' + b}$. (89)

If d and d' are measured from the outside of the flange, eq. (89) will also give x , measured from the same line.

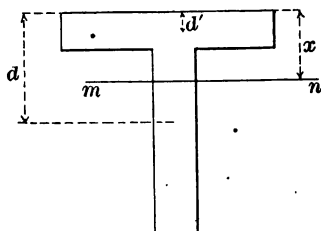


Fig. 42

Supposing the coefficients of elasticity for tension and compression to be equal, in which case the neutral axis does pass through the center of gravity of the section, it is evident that the greatest strain would be on the end of the stem, and that the value of x , found from Fig. 41, would be the proper value to be used.

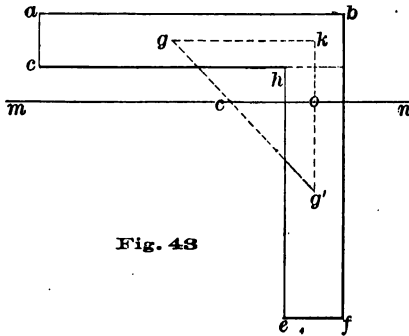


Fig. 43

It is evident that (89) applies without change to finding the position of the axis mn , through the center of gravity of angle irons, parallel to a leg, whether for equal or unequal legs. The position of this axis is all that is required in practice.

If, for any reason, the *center of gravity* is required, it may be easily found as follows:

Find mn as above; then observing that the centers of gravity, g and g' , of the legs are at their geometrical centers, join

g and g' . The intersection c with mn determines c , the center of gravity.

Or, let a = area of upper leg, and a' that of the lower. Take c such that

$$\frac{cg}{cg'} = \frac{a'}{a} \text{ or } \frac{cg}{gg'} = \frac{a'}{a+a'} \text{ or } cg = \frac{a'}{a+a'} \times gg' \quad (90)$$

This determines the position of c . Drawing gk and $g'k$ parallel to ab and bf we have:

$$\frac{Ok}{kg'} = \frac{cg}{gg'} = \frac{a'}{a+a'} \text{ or } Ok = \frac{a'}{a+a'} kg' \quad (91)$$

This determines the position of the axis mn , with reference to k and g' and therefore with reference to ab and ef .

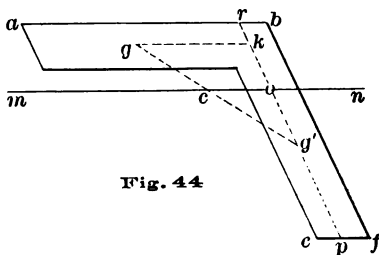


Fig. 44

For an obtuse angle iron, it is evident that the line gg' will be divided at c and therefore the line kg' at o in the same ratio as though the angle was right; and since the line kg' is of the same length as for a right angled "angle," ko is also the same. This gives an easy way of finding the position of mn .

The perpendicular distance of mn from ab and ef , are equal respectively to $or \sin gkp$ and $op \sin gkp$.

To find the axis through the center of gravity of a channel parallel to the web, consider each flange divided into a rectangle and a triangle as shown:

Let t = thickness of the web,

h = height of a flange,

x = distance from back of channels
to the center of gravity of the
channel,

b = area of the web,

a = area of the rectangular portion
of a flange,

a' = area of the triangular portion of
a flange,

and A = the total area of the channel.

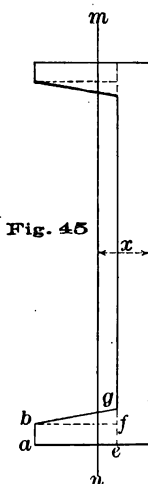


Fig. 45

Taking moments about the back of the channel we have,

$$(2a + 2a' + b)x = b \times \frac{1}{2}t + 2a(t + \frac{1}{2}h) + 2a'(t + \frac{1}{8}h),$$

$$\text{or } x = \frac{t(2a + 2a' + \frac{1}{2}h) + h(a + \frac{2}{3}a')}{2a + 2a' + b} \quad (92)$$

In a similar manner we easily find the center of gravity of all practicable forms.

DECK BEAMS.

To find the moment of resistance about any axis.

If the beam is of the form shown in Fig. 46 which is the common one, eq. (65), will give the moment of the two

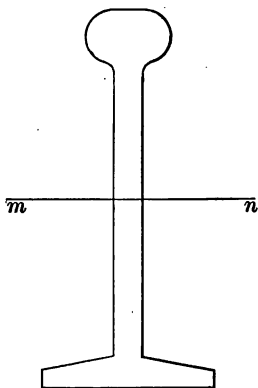


Fig. 46

semicircles or circle; the formula for number 2 of Table I. the same of each part of the web above and below the axis; and eq. (55) of the flange.

If the beam is of the form shown in Fig. 47 eq. (68) or (69) will apply to the upper semicircle and eq. (70) or (71)

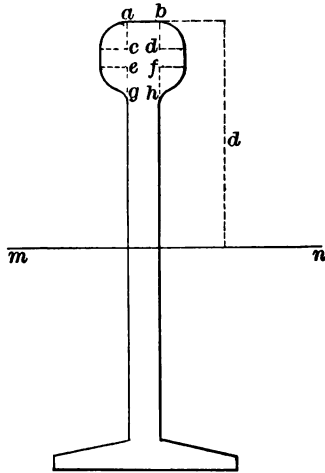


Fig. 47

to the lower, etc. It is easy to see, however, that the lever arm of the head as a whole, is in value between that of a circle whose diameter is $ag=h$ (say), and that

vertical stems, consider each divided into triangles by diagonals *be*, etc. Then from the formulas for numbers 3 and 4 of Table I.

$$M_1 = 2 \left\{ \frac{1}{12} T' \left(\frac{D}{2} \right) + \frac{1}{4} \left(\frac{D}{2} \right)^2 \right\} = \frac{1}{24} D^2 (3t + T') \quad (93)$$

Since the moment of the horizontal stem is very small compared with the vertical stem, we may consider the former to be flat, as was done in treating the flanges of the eye beams in Fig. 35. We thus find the moment of resistance of the horizontal stem to be

$$M_2 = \frac{1}{48} \frac{D - T}{D} (t + T)^2 \quad (94)$$

(93) and (94) give for the total moment

$$M = \frac{1}{24} D^2 (3t + T') + \frac{1}{48} \frac{D - T}{D} (t + T)^2 \quad (95)$$

$$\text{or } M = \frac{1}{24} D^2 (3t + T') + \frac{1}{48} (t + T)^2, \text{ nearly.} \quad (96)$$

These represent to the best advantage the usual formulas for this case. We will, however, derive a formula twice as short and three times as accurate.

We notice that the area cut off, in rounding the four corners at e , f , E and F is a little less than the difference between the areas of a square and a circle, whose diameters are each equal to t .

$$\text{This difference is} = t^2 - \frac{\pi}{4}t^2 = .2146t^2.$$

The lever arm of this area is a little less than $\frac{D}{2}$. Hence the moment is a little less than

$$.2146t^2 \frac{D}{2} = .1073t^2D \text{ or } M' = \frac{1}{16}t^2D, \\ \text{very nearly. (97)}$$

The total moment is, therefore,

$$M = \frac{1}{24}D^2(3t + T') - \frac{1}{16}Dt^2 + \frac{1}{48}\frac{D-T}{D}(t+T)^2 \\ \text{(98)}$$

In regard to eq. (98), we observe that the term $\frac{1}{16}Dt^2$ is usually and properly disregarded; and that the term $\frac{1}{48}\frac{D-T}{D}(t+T)^2$, as well as the quantity $\frac{1}{24}D^2(T'-T)$ is in practical cases much smaller, and both together smaller than $\frac{1}{16}Dt^2$. Hence

$$\frac{1}{10}Dt^2 - \frac{1}{24}D^2(T' - T) - \frac{1}{48}\frac{D - T}{D}(t + T)^2$$

= 0, very nearly.

Adding this to (98), we obtain the moment of resistance

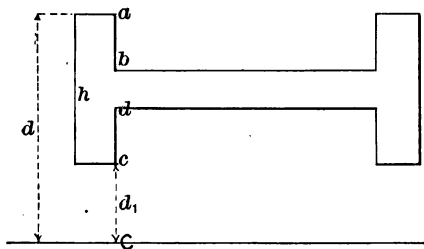
$$= M = \frac{1}{24}D^2(3t + T) \quad (99)$$

This is believed to be the simplest and most accurate approximate formula possible to deduce. The following table will show that, taking all the forms in Carnegie's book as a test, eq. (99) is three times as accurate as (96), or as the ostensibly accurate results (and they are sufficiently accurate) given in Carnegie's "Pocket Companion:"

Size.	Thickness in inches at the end and root of flange.	Eq. (96) or Carnegie's.	True moment eq. (98).	Eq. (99).
4 × 4	$\frac{1}{8}$	1.16	1.11	1.13
3½ × 3½	$\frac{1}{8}$	0.85	0.80	0.83
3 × 3	$\frac{1}{8}$	0.55	0.52	0.53
2½ × 2½	$\frac{1}{8}$	0.36	0.34	0.35
2 × 2	$\frac{1}{8}$	0.20	0.19	0.19
1½ × 1½	$\frac{1}{8}$	0.087	0.083	0.082

To find the moment of resistance of an eye beam, the neutral axis being any line parallel to the center line of the web.

Fig. 49



Let $ac = h$, $bd = t$, $Cd = d_1$.

The lever arm of the flanges is by eq. (55)

$$l = d_1 + \frac{1}{8} \frac{h^2}{d} = d_1 + .33 \frac{h^2}{d} \quad (55)$$

To find the lever arm of the web. Supposing the strain at b equal to unity, the same equation would give the lever

arm $= d_2 + \frac{1}{8} \frac{t^2}{(d_2 + t)}$. But the strain at b is

$\frac{bC}{aC} = \frac{d}{d_2 + t}$, and hence the lever arm $= l'$

$$\begin{aligned}
&= \left(d_1 + \frac{t^2}{3(d_1 + t)} \right) \frac{d_1 + t}{d} = \frac{d_1^2 + d_1 t}{d} + \frac{t^2}{3d} = \\
&\frac{(d_1 + \frac{1}{2}t)^2}{d} + \frac{t^2}{12d} = \frac{(d - \frac{1}{2}h)^2}{d} + \frac{t^2}{12d} = d - h \\
&+ \frac{h^2}{4d} + \frac{t^2}{12d} = d_1 + \frac{h^2}{4d} + \frac{t^2}{12d} = d_1 + \frac{h^2}{4d}, \\
&\text{very nearly, (55')}
\end{aligned}$$

We observe that $\frac{t^2}{12d}$ will not amount, in average cases, to nearly one hundredth part of $\frac{h^2}{4d}$, or to scarcely a perceptible part of the lever arm.

When $t=h$ (55') gives $l' = d + \frac{h^2}{4d} + \frac{h^2}{12d} = d_1 + \frac{h^2}{3d} = l$ as it ought. An arithmetical mean of the above lever arms used for both stem and flanges will give results sufficiently accurate in almost all cases.

$$\begin{aligned}
&\text{This lever arm is, } l = d + \frac{1}{2} \left(\frac{h^2}{3d} + \frac{h^2}{4d} \right) = d_1 \\
&+ \frac{7h^2}{24d} = d_1 + .29 \frac{h^2}{d} \text{ (55'')} = d_1 + .3 \frac{h^2}{d} \\
&\text{nearby, (55''')}
\end{aligned}$$

If a = area of each flange and b that

of the web, we have from (55) and (55'), for the beam,

$$\begin{aligned} M_1 &= 2a \left(d_1 + \frac{h^2}{3d} \right) + b \left(d_1 + \frac{h^2}{4a} + \frac{t^2}{12d} \right) \\ &= 2a \left(d_1 + \frac{h^2}{3d} \right) + b \left(d_1 + \frac{h^2}{4a} \right) \text{ very nearly (100)} \\ \text{or from (55''), } M_1 &= (2a + b) \left(d_1 + .3 \frac{h^2}{d} \right) \end{aligned}$$

CHANNEL BARS.

Let $ac=h$, $bc=t$ and an (n is at the middle of bc) $=h_1$, etc. The lever arm of the flanges, as we have just seen, is $l=d_1 + \frac{h^2}{3d}$. Observing that, in reference to the web, $\frac{1}{2}h$ in Fig. 49, is the same as h_1

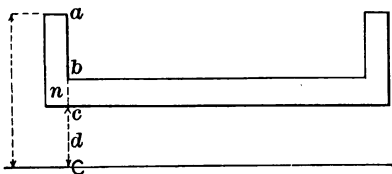


Fig. 50

in Fig. 50, eq. (55) gives for the lever arm of the web

$$l' = \frac{(d-h)^2}{d} + \frac{t^2}{12d} = \frac{(d-h)^2}{d} \text{ very nearly (102)}$$

The notation being as before, the moment is therefore

$$= M_1 = 2a \left(d_1 + \frac{h^2}{3d} \right) + b \frac{(d-h)^2}{d} \quad (103)$$

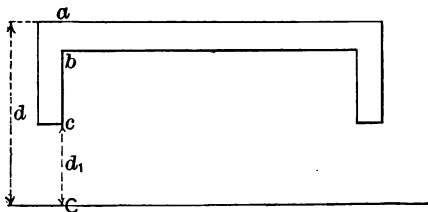


Fig. 51

In Fig. 51 let $ac=h$, $bc=t$, etc. Eq. (55) gives the lever arm of the flanges,

$$= l = d + \frac{h^2}{3d};$$

and the lever arm of the web

$$= l'' = d - t + \frac{t^2}{3d} = d - t, \text{ very nearly.}$$

$$\text{Hence } M_1 = 2a \left(d_1 + \frac{h^2}{3d} \right) + b \left(d - t + \frac{t^2}{3d} \right)$$

$$= 2a \left(d_1 + \frac{h^2}{3d} \right) + b (d - t) \text{ very nearly (104)}$$

These formulas evidently apply without change to angle irons, as shown in Fig. 52.

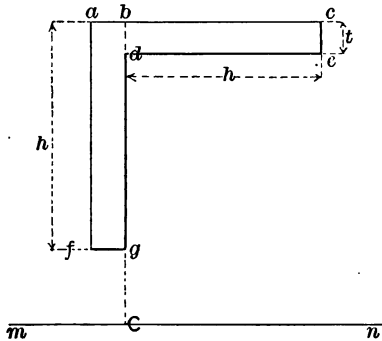


Fig. 52

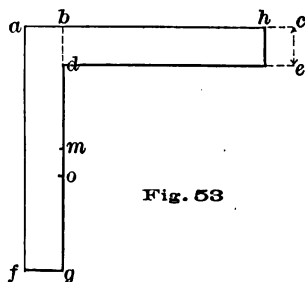
We shall now apply the preceding formulas to finding the moment of resistance of compound beams, girders, etc., selecting our examples for the purpose from actual practice. We shall deduce formulas exceedingly short and practically perfectly accurate. We will first consider angle irons.

Referring to Fig. 52 the lever arm of

the vertical leg is equal to $d-h+\frac{h^2}{3d}$, and of the horizontal leg to $d-t+\frac{t^2}{3d}=d-t$ very nearly.

Hence, supposing $bc=af=h$, we have the lever arm of the angle equal to an arithmetical mean between the lever arms of the legs $=l=\frac{1}{2}\left(d-t+d-h+\frac{h^2+t^2}{3d}\right)$
 $=d-\frac{1}{2}(h+t)+\frac{h^2+t^2}{6d}$ (105); $=d-\frac{1}{2}(h+t)$

nearly, which shows that the extremity of the lever arm, or the center of moments, is in this case nearly at (though a little above) the middle of dg .



Suppose, now, the horizontal leg to be shortened an amount $hc=ab=t$, so that $ah=af=h$, the legs being equal: to find the decrease in the lever arm of the angle. M , the middle point of dg , is the center of moments, very nearly, for the angle acf ; and o the same for the angle ahf . Let $mo=x$.

Taking moments about m , and observing that the areas of the parts of the angle are as their lengths, we have:

$$mo(af+bh)=do \times hc, \text{ or } x(2h-t)=\frac{1}{2}(h-t)t$$

$$\therefore x=\frac{t(h-t)}{4h-2t} < \frac{t(h-t)}{4h-4t}=\frac{1}{4}t, \quad \text{or} \quad x=\frac{1}{4}t$$

nearly. Subtracting this from (105), we have for equal legs—

$$\text{Lever arm} = l' = d - \frac{1}{2}(h+t) - \left(\frac{t}{4} - \frac{h^2+t^2}{6d} \right) \left(\frac{90}{1} \right)$$

or effective depth

$$=D' = D - (h+t) - \left(\frac{t}{2} - \frac{h^2+t^2}{3d} \right) \quad (106)$$

Now, suppose the vertical leg $=h$ and the horizontal leg $=h+e$, then as above we readily find,

$$x = \frac{e(h-t)}{4h+2e} < \frac{eh}{4h+2e} < \frac{eh}{4h} = \frac{1}{4}e.$$

Applying this to all the forms rolled by Carnegie, taking average thickness of legs, we find the average to be

$$x = .172e = \frac{1}{6}e \text{ very nearly.}$$

Hence for unequal legs the lever arm is,

$$l'' = d - \frac{1}{2}(h + t) - \left(\frac{t}{4} - \frac{h^2 + t^2}{6d} \right) + \frac{1}{6}e \quad (105'')$$

And the effective depth is

$$D'' = D - (h + t) - \left(\frac{t}{2} - \frac{h^2 + t^2}{3d} \right) + \frac{1}{3}e \quad (107)$$

$\frac{t^2}{3d}$ is too small to take note of.

Again, $\frac{h^2 + t^2}{3d}$, is only a small part of unity, and furthermore, in no properly proportioned beam can this expression differ from $\frac{t}{2}$ by an appreciable amount—by an amount scarcely affecting the decimal places of the result. This becomes apparent when we reflect that, approximately, $t \propto h \propto D$, and hence

$$\frac{h^2 + t^2}{3d} \text{ or } \frac{h^2}{3d} \propto \frac{d^2}{d} \propto d \propto D, \text{ and } \frac{t}{2} \propto \frac{h^2 + t^2}{3d}.$$

For an average case, let $t = \frac{3}{8}$, $h = 3$, $D = 30$, we have

$$\frac{t}{2} \cdot \frac{h^2}{3d} = \frac{3}{16} - \frac{1}{8} = \frac{1}{80} = .012.$$

The equations become, therefore, for equal legs, $D' = D - (h + t)$, (108)

And for unequal legs,

$$D'' = D - (h + t) + \frac{1}{8}e. \quad (109)$$

In case of unlike angles for the upper and lower flanges we have the effective depth

$$= D'' = D - \frac{1}{2}(h + h' + t + t') + \frac{1}{8}(e + e') \quad (110)$$

in which the accented letters refer to the angles on one of the flanges.

If $t = t'$, effective depth

$$= D'' = D - \frac{1}{2}(h + h' + 2t) + \frac{1}{8}(e + e') \quad (111)$$

PLATE GIRDERS.

Let Fig. 54 represent a single plate girder formed by a plate and four angles.

In proportioning built beams, it is understood to be the duty of the web to maintain a rigid connection between the

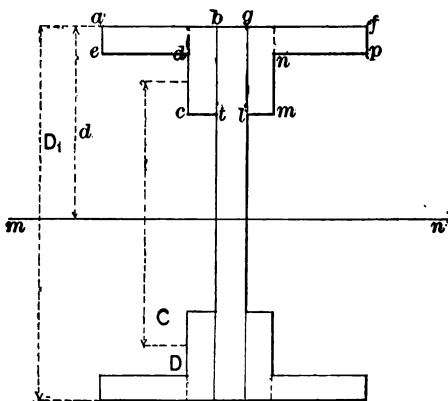


Fig. 54

flanges; and it is usual to assume that "the flanges sustain all the horizontal strains and no other, and that the web sustains only the vertical strains."

Some engineers treat the web, as also resisting horizontal strains, the same as a beam with no flanges; and since it can only partially do so, they attempt to correct the assumption by allowing a proportionately smaller strain per $\square$ of cross section. But this is plainly un-

philosophical, since the ratio of the area of the web to that of the flanges is not constant. If anything at all is allowed for the web in resisting horizontal strains, such a part only of its resisting power as a beam, should be allowed as experiment will justify.

The flanges are frequently made equal, the supposition being, however, that the resisting power of the cross section of the compression flange in such cases, is somewhat greater than that of the net area (rivet holes deducted) of the tension flange.

I will in the following examples, give the results by my formulas, as well as those by the usual formulas, noting the errors of the latter.

Example 1. Web $20'' \times \frac{1}{4}''$; 4 angles $2\frac{1}{4}'' \times 2\frac{1}{4}'' \times \frac{1}{4}''$, as shown in Fig. 54. Instead of multiplying the area of each flange by its lever arm, we may multiply the area of one flange by twice its lever arm, or the "effective depth" of the beam.

Now, by (108), the effective depth $= D_1 = 2d - (h + t) = 20 - 2\frac{1}{2} = 17.5$. And

the area of the two angles is equal to $a=2\frac{1}{8}$.

Hence the moment by 77 is equal to $D_1a=17.5 \times 2.12=37.10$.

The true moment found by equation (106) is 37.28.

Taking the effective depth equal to the clear depth between the horizontal legs of the angles, which is the least value used in general practice, and, therefore, leads to the *least* error, we find the moment $=19.5 \times 2\frac{1}{8}=41.44$, showing an error of 4.08, or about 11%.

Example 2. Web $27'' \times \frac{1}{4}''$; 4 angles $3'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$.

The effective depth by (109)

$$=D-(h+t)+\frac{1}{8}e=27-2\frac{3}{4}+\frac{1}{8}=24.42.$$

The area of two angles $=2.62$; and $24.42 \times 2.62=63.98$. -

The true moment is 64.14.

Example 3. The floor beams of a bridge are $17\frac{3}{8}$ feet apart and $17\frac{1}{8}$ feet long from center to center of pins; and fully stiffened so that their length need not be regarded when considering the strains upon them. Roadway, 16 feet

wide in the clear. Rolling load, 80lb per
 □ foot, or 1280lb per lineal foot of bridge.

A beam, web $20'' \times \frac{1}{4}''$ and 4, $3'' \times 2''$,
 4lb angles ($\frac{1}{4}$ inch thick, area 1.19 □'') is
 proposed. Iron supposed to resist 8,000
 pounds per square inch in compression
 and 10,000 in tension. Is the beam suf-
 ficiently strong?

1° The dead load per lineal foot of
 bridge is:

Flooring (oak),	Pounds.
$2\frac{1}{2} \times 16 = 40$ ft. ; $40 \times 4\frac{1}{2} = 180$	
11 joists (pine),	
$\frac{3 \times 12 \times 11}{12} = 33$ ft. ; $33 \times 3\frac{1}{8} = 110$	
2 felloe guards,	
$\frac{4 \times 6 \times 2}{12} = 4$ ft. ; $4 \times 3\frac{1}{8} = 13$	
	Total, 303

Rolling load, $16 \times 80 = 1280$ pounds.

Total per lineal foot, 1583 “

$1583 \times 17\frac{3}{8} = 27263 =$ the external load on
 the beam.

$$\text{Area of web} = 20 \times \frac{1}{4} = 5 \text{ sq. in.}$$

$$\text{and weight} = \frac{5 \times 10}{3} = 16.67$$

$$\text{Weight of angles per foot} = 4 \times 4 = 16.00$$

$$\text{Total, } \underline{32.67}$$

Add for stiffening angles,

$$\text{filling plates and rivets, } 20\% = 6.53$$

$$\underline{39.20}$$

$$\text{Hence weight of beam} = 39.2 \times 17\frac{1}{2} = 673$$

$$\text{Add, } \underline{27263}$$

$$\text{Total load} = \underline{27936}$$

The moment of the load

$$= m = \frac{27936 \times 17\frac{1}{2} \times 12}{8} = 719352.$$

The compressive side of the beam is the weaker.

The effective depth of angles is by (109), $D_1 = 20 - 2\frac{1}{4} + \frac{1}{8} = 18\frac{1}{8}$, hence, *considering the web fully effective*, equation (76) gives for the moment of resistance of the beam;

$$m' = 8000(18\frac{1}{8} \times 2.38 + \frac{1}{8} \times 5 \times 20) = 477630$$

Now $\frac{477630}{719352} = 66$ nearly; and we find that the beam has at most, *according to*

the specifications, about two-thirds of the required strength or capacity.

2° Disregarding the coefficient 8,000, and considering the tensile side to be the weaker. Using $\frac{5}{8}$ " rivets in $\frac{1}{4}$ " holes, the angles being $\frac{1}{4}$ " thick, the area lost by a rivet in each angle is $2 \times \frac{1}{4} \times \frac{1}{4} = .344$ ".

Hence the effective area of a flange is $2.375 - .344 = 2.03$ ", and we have moment of resistance of beam

$$= 10000(2.03 \times 18\frac{1}{2} + \frac{5}{8} \times 20) = 533792.$$

The beam has, under *this* supposition, a capacity of $\frac{533792}{710000} = 74\%$ of that required.

3° Computing, according to the Carnegie formula, coefficients and all, which formula was professedly (though in view of the 8000 coefficient, inconsistently) used in this case; we have the moment of resistance of beam equal to

$$10000 \times 2.38 \times 20 = 476000.$$

Now, $\frac{476000}{710000} = .66$, showing that the beam has over 66%, or about $\frac{2}{3}$ of required capacity.

4° Referring to the Carnegie table we

find the area required for each flange = 3.58 square inches, and as we have an area equal to 2.38 square inches, the capacity of beam is $\frac{2.38}{3.58} = .66$, or 66% as above. We will try a web plate $\frac{1}{4}'' \times 24''$.

Flanges, four, $3'' \times 2'' \times \frac{5}{16}''$ (5lb angles).

$$\text{Weight of web} = \frac{\frac{1}{4} \times 24 \times 10}{3} = 20$$

$$\text{Weight of angles} = 4 \times 5 = 20$$

$$\begin{array}{r} 40 \\ \text{Add } 20\% \quad 8 \\ \hline 48 \end{array}$$

$$\text{Hence weight of beam} = 48 \times 17\frac{1}{8} = 824$$

$$\text{Add } 27263$$

$$\text{Total load } 28087$$

The moment of the load

$$= m = \frac{28087 \times 17\frac{1}{8} \times 12}{8} = 723240.$$

The effective depth of flanges

$$= 24 - (2 + \frac{5}{16}) + \frac{1}{8} = 22.02.$$

$$\text{Hence } m' = 8000(aD_1 + \frac{1}{8}BD)$$

$$= 8000(2.98 \times 22.02 + 24) = 717040.$$

The beam is therefore about one-half of one per cent. only short of the capacity required, and is therefore satisfactory.

According to Carnegie we have moment of resistance = $10000 \times 2.98 \times 24 = 715200$, showing a deficiency of about 1%.

A plate girder with flange plates. The effective depth of the flange plates is but little greater than the web plate D .

To find the effective depth of the angles we observe that were the strain at c unity, the effective depth by (109) would be $D - (h + t) + \frac{1}{8}e$. But the strain at c is equal to

$$\begin{aligned}\frac{mc}{ma} &= \frac{d}{d+t_1} = \frac{D}{D+2t_1} = \frac{D-2t_1}{D} \text{ nearly.} \\ &= 1 - \frac{2t_1}{D} \text{ nearly.}\end{aligned}$$

Hence the effective depth is equal to

$$\begin{aligned}D'' &= [D - (h + t) + \frac{1}{8}e] \left(1 - \frac{2t_1}{D}\right) = D - (h + t) \\ &\quad + \frac{1}{8}e - 2t_1 + \\ &= D - (h + t + 2t_1) + \frac{1}{8}e \text{ nearly. (109')}\end{aligned}$$

For equal legs, $e=0$, and we have effective depth

$$=D'=D-(h+t)-2t_1=D-(h+t+2t_1) \quad (108')$$

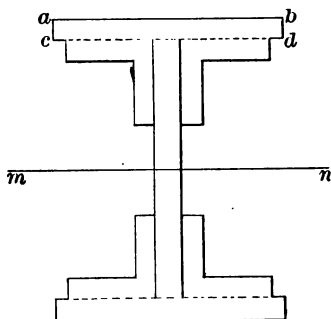


Fig. 55

Eqs. (108') and (109'), compared with (108) and (109), show that the reduced effective depth of a section covered by a plate is approximately equal to the effective depth without the plate, less the thickness of the plate. Of course the principle applies to all sections situated substantially the same.

For unequal legs, with plates whose thickness is t_1 , eqs. (110) and (111) become,

$$D'' = D - \frac{1}{2}(h + h' + t + t') + \frac{1}{8}(e + e') - 2t_1 \quad (110')$$

$$\text{and } D'' = D - \frac{1}{2}(h + h' + 2t) + \frac{1}{8}(e + e') - 2t_1 \quad (111')$$

For one plate we write in those several equations, t_1 , for $2t_1$. The effective depth is to be used with the weaker flange.

Example—A track stringer (17 feet and one inch long from center to center of floor beam).

Web, $21\frac{1}{2}'' \times \frac{5}{16}'' \times 16' 11\frac{1}{2}''$ long.

Upper flange, 2 angles $3\frac{1}{2}'' \times 3'' \times \frac{3}{8}''$,
one plate, $8'' \times \frac{3}{8}''$.

Lower flange, 2 angles, $5'' \times 4'' \times \frac{3}{8}''$.

(111') gives, effective depth

$$= 21.5 - \frac{1}{2}(3 + 4 + \frac{5}{4}) + \frac{1}{8} \times \frac{3}{2} - \frac{3}{8} = 17.5.$$

The lower flange is the weaker.

Area of lower flange is $6.47 \square''$.

Hence the moment is $6.47 \times 17.5 = 113.22$.

Were the depth of the web taken for the effective depth, the error would be,

$$\frac{21.5 - 17.5}{17.5} = 23\% \text{ nearly.}$$

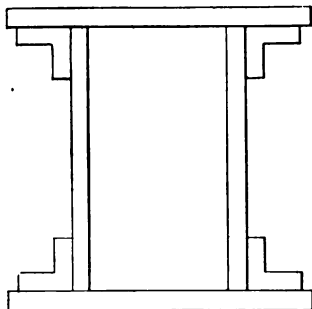


Fig. 56

A box girder (Fig. 56) may be treated precisely as a plate girder, the former having two web plates, the latter one.

A, box girder as shown in Fig. 57.

D being the depth of channels;

D_1 average depth between flanges;

b the area of the webs;

a the area of flanges on one side of the axis;

t average thickness of flange;

A the area; and

t_1 the thickness of a plate.

Eq. (55) gives the effective depth of plates but little greater than D ; and the

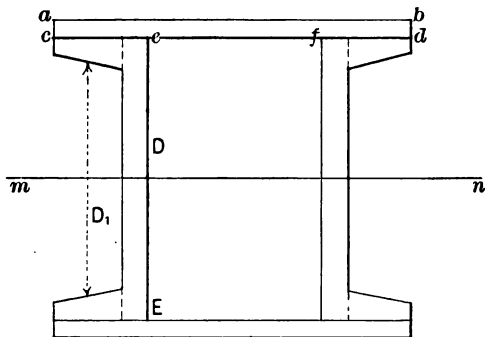


Fig. 57

same eq. observing the remark under eqs. (108') and (109') shows the effective depth of the web to be but little greater than $D - 2t$, and that of the flanges a little greater than $D_1 - 2t_1 = D - 2t - 2t_1$.

The effective depth for the channels and plates together will not vary much from $D - 2t_1$.

Hence the moment

$$M = (u + A + \frac{1}{8}b) (D - 2t_1) \quad (112)$$

Example—15" channels.

Let plate be $12'' \times \frac{1}{2}''$;

Area of two flanges=4.125.

Area of two webs=15.75 and $D-2t_f=14$.

Hence $m=14(6+4.125+2.62)$

$$=14 \times 12.75=178.5.$$

If the channels are turned inward the moment will, of course, be the same.

In a similar way may the effective depth and moment of resistance of any form of girders be found.

SOLID EYE BEAMS.

Example—Floor beams $17\frac{1}{2}$ feet apart and $17\frac{1}{8}$ feet from center to center of pins. Roadway 16 feet wide in the clear. Rolling load 1280 pounds per lineal foot of bridge. Allowable strain 10,000 pounds per square inch.

A 12", 42 pound eye beam (Carnegie No. 3) is used. Is it sufficient for the load? The moment of the beam is $45.96 \times 10000=459600$.

Load per lineal foot of bridge;

Flooring (pine 3")

$$3 \times 16 = 48 \text{ ft.}, 48 \times 3\frac{1}{2} = 160 \text{ pounds.}$$

9 joists (pine 3" $\times$ 12")

$$3 \times 9 = 27 \text{ ft.}, 27 \times 3\frac{1}{2} = 90$$

2 wheel guards (pine 6" $\times$ 6")

$$2 \times 3 = 6 \text{ ft.}, 6 \times 3\frac{1}{2} = 20$$

$$\underline{270}$$

Add 1280

$$\underline{1550}$$

Load on the beam

$$= 1550 \times 17\frac{1}{2} = 27125 \text{ pounds.}$$

$$\text{Weight of beam} = 17\frac{1}{2} \times 42 = \underline{721}$$

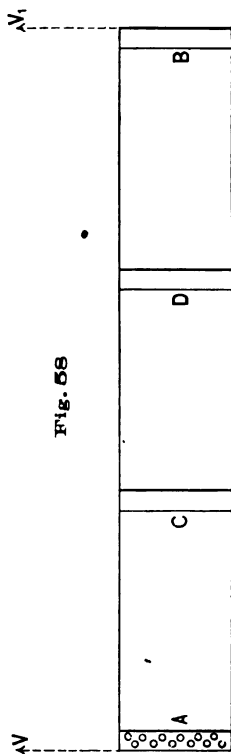
$$\text{Total load on beam } 27846$$

The moment of the load

$$= \frac{27846 \times 17\frac{1}{2} \times 12}{8} = 717034.$$

Now $\frac{717034}{1117000} = .64$. This shows that the floor beam has but 64% of the required capacity. A 15", 50lb beam has a moment of resistance = $70.6 \times 10000 = 706000$, and has therefore a capacity of $\frac{717034}{717000} = .98$, or 98% of that required, and may be considered sufficient.

Complete design for a plate floor beam



with cover plates for a single track railway bridge.

Let the beam be 15 feet between supports, total length 16 feet; and let $AC = DB = 5\frac{1}{8}$ feet = 62 inches and $CD = 4\frac{3}{8}$ feet = 56 inches. The ends of the track stringers rest at C and D.

Let the web plate be 28 inches deep and $\frac{3}{8}$ of an inch in thickness. The flanges alone are supposed to resist the longitudinal strains of compression and tension.

We will suppose the beams to be 16 feet apart, and the wheel spaces 7 feet; and that 24,000 pounds rest upon each pair of wheels. The train weight on a beam is therefore

$$24,000 + 2 \times \frac{2}{18} \times 24,000 + 2 \times \frac{2}{18} \times 24,000 \\ = 57,000 \text{ pounds, or } 28,500 \text{ at C and at D.}$$

We will assume the weight of beam to be 100 pounds per foot, or 1,600 pounds; and that of the track stringer 80 pounds per foot, or 1,280 pounds. Hence the external weight on the beam at C or at D is

$$28,500 + 640 = 29,140 \text{ pounds.}$$

The reaction V at A, is equal to

$$29,140 + 800 = 30,000 \text{ pounds nearly.}$$

The weight of the beam between A and C is $5\frac{1}{8} \times 100 = 520$ pounds.

$$\begin{aligned} \text{Hence the bending moment at C is} \\ 30,000 \times 62 - 520 \times 31 = 1,844,000 \\ \text{inch pounds nearly.} \end{aligned}$$

To proportion the flanges for the middle portion of the beam.

We will try $3''\frac{1}{2} \times 3'' \times \frac{1}{2}''$ angle irons (10.4 lb., $3.11 \square''$) with a plate $9'' \times \frac{5}{16}''$.

We will use $\frac{3}{4}''$ rivets in both legs of the angles:

Area of two angles is	6.22 $\square''$
Area lost by a rivet, is $2 \times \frac{3}{4} \times \frac{1}{2}$	.75''
	5.47''
Net area of plate is $(9 - 2 \times \frac{3}{4}) \times \frac{5}{16}$	2.34''
	7.81''

For an approximate result we may assume the effective depth of plate and angles together to be equal to the dis-

tance between the lines of centers of rivets in the vertical legs of the angles.

This is approximately equal to $D-h$.
In this case $D-h=28-3=25$.

We will take the allowable tensile and compressive resistances in the flanges to be 10,000 pounds per square inch.

Hence, moment of resistance

$$=10,000 \times 7.81 \times 25 = 1,952,500.$$

This is about 110,000 or 6% in excess of requirements.

We will therefore try the above angle-irons, with a plate $8'' \times \frac{5}{16}''$, computing more carefully

	Square Inches.
Net area of angles	= 5.47
Net area of plate = $6\frac{1}{2} \times \frac{5}{16}$	= 2.03
Total,	<u>7.50</u>

The lever arm of angles

$$= 28 - (3 + \frac{1}{2} + \frac{5}{8}) + \frac{1}{8} = 24.04 \text{ inches,}$$

and that of the plate = 28 “

Hence moment of resistance

$$\begin{aligned} &= 10,000 (5.47 \times 24.04 + 2.03 \times 28) \\ &= 10,000 (131.5 + 56.8) = 1,315,000 \\ &\quad + 568,000 = 1,883,000 \text{ inch pounds.} \end{aligned}$$

This is about 2% in excess of requirements, and may be considered satisfactory.

To find the number of rivets required in the vertical legs of the angle irons.

The total strain in the flanges must pass through these rivets. Hence, assuming the rows of rivets to be 25 inches apart, as above, the flange strain at C is

$1,844,000 \div 25 = 73,760$ pounds,
or 36,880 pounds on each angle.

The bearing value of a $\frac{3}{4}$ " rivet through a $\frac{3}{8}$ " plate is, by Table IV., 4,220 pounds. Hence number required is $\frac{73,760}{4,220} = 18$.

Since the rivets sustain double shear in the flanges, Table IV. shows that the resistance of the rivets to shearing is much greater than to bearing, and that the former need not therefore be considered.

Now, the lever arm of the strain on the flanges is $\frac{1}{2} (\frac{3}{8} + \frac{1}{2}) = \frac{7}{16}$ ", and the moment, $36,880 \times \frac{7}{16} = 16,135$ inch pounds. The moment of a rivet is 780 inch pounds. Hence number of rivets required to resist bending is $\frac{16,135}{780} = 21$.

We will insert, say, 24 rivets in the 62 inches from A to C, spacing about $2\frac{1}{2}$ inches apart.

The Table shows that with a $\frac{3}{8}$ " plate the double shearing is equal to the bearing resistance for $\frac{1}{2}$ " rivets and over, and we have just found that the bending is nearly equal to the bearing resistance for $\frac{3}{4}$ " rivets and over.

Now, since the bearing resistance increases as the product of the diameter of rivet and thickness of plate, and the shearing resistance as the square of the diameter of the rivet, and the bending resistance as the quotient of the cube of the diameter of the rivet, divided by the thickness of the plate and the angle leg; and since the thickness of the plate and that of the angle legs increase approximately with the diameter of the rivets, it is evident that the three resistances increase or decrease approximately together.

With the plates and angles used, however, it will be found that in general, the smaller rivets are weaker for bending

than bearing and the larger rivets stronger.

It is unnecessary to compute the strain at any point between C and D, for it would differ from the strain at C or D by an amount due to the weight of the beam only, which is very small.

To find the length of the cover plates.

Since the weight of the beam is small compared with the load, the flange stress or moment of flange stress increases almost uniformly from A to C.

Hence an increased section is first needed at $(5.47 \div 7.50) \times 62 = 45$ inches from A.

If, however, the end of the cover plate is placed at the above point, the angles will be strained to the allowable limit at that point, the stress will decrease quite rapidly on either side of it, and the beam will be somewhat distorted in consequence.

It is better, therefore, to extend the end of the plate from the point in question one-fourth or one-third of the distance to A.

We will use a cover plate 10 feet long, reaching from a point $2\frac{1}{2}$ feet from A to a point $2\frac{1}{2}$ feet from B.

We will use $\frac{3}{4}$ " rivets to bind the cover plate to the angles, and they should, so far as possible, be pitched half way between the rivets in the other leg of the angle irons.

It is customary to extend the cover plate, say, one and a half times its width, but the proper distance to extend the cover plate does not so much depend upon its width as upon the space between its end and the end of the beam, as above pointed out.

We have so far confined our attention to the tension flange, but the same design is surely sufficient for the compression flange, since the resistance of wrought-iron for compression is fully equal to that for tension, and the rivet holes in the compression flange need not be deducted.

The upper (compression) flange is frequently constructed with a cover plate, the lower flange with heavier angle irons, without a cover plate.

While less material is lost in the tension flange by rivet holes, without than with a cover plate, the use of a cover plate above and none below, leaves the flanges unsymmetrical. Besides, the effective depth is less without than with cover plates, and hence the advantage of the above arrangement is not so great as might be supposed. For example, the gross material in the above flange is $6.22 + 2.50 = 8.72$ square inches.

Gross material in two angle irons,

$4'' \times 3\frac{1}{4}'' \times .60''$, is..... 8.72 sq. ins.

Area lost by a rivet is $.2 \times .60 \times \frac{3}{4}$. .90 " "

Net area of angles = 7.82 " "

Effective depth of angles

$$= 28 - (3\frac{1}{2} + .60) + .17 = 24.07''$$

Now, $7.82 \times 24.07 \times 10,000 = 1,882,274$, which is just about the same as for the flange found above. It is generally best to use a cover plate on each flange or none at all.

To find the number of rivets in the cover plate.

The total stress carried by the cover plate is $2.03 \times 10,000 = 20,300$ pounds.

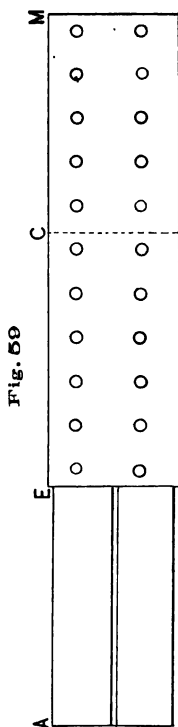
The bearing value of a rivet is 4,220 pounds. Hence, the number of rivets required between the end of the plate and the point C is

$$\frac{20300}{4220} = 5.$$

Fig. 59 shows the left half of the beam AEM, with the half cover plate EM, AE = 30, EC = 32, and CM = 28 inches. Placing rivets 2 inches from the end of the plate and 4 inches from the middle, making the three spaces on the left 3, 4 and 5 inches respectively, and the others 6 inches, the plan of the beam will appear as in the figure. This gives more than double the number required between the end of the plate E and C, and plenty between C and M.

In case more than one cover plate is used, there must be rivets enough between the ends of the first and the second plate to take up the tension in the first plate.

The number is found precisely as above.



In the compression flange, the cover plate between any two consecutive rivets forms a solid rectangular column.

And hence the rivets should not be so far apart as to allow of any material amount of long column flexure.

The pitch of the rivets should probably not be more than about 20 times the thickness of the plate.

The reaction at the ends of the beam is 30,000 pounds.

The transverse section of the web is $28 \times \frac{3}{8} = 10.5$ square inches, the shearing resistance of which is $10.5 \times 8,000 = 84,000$ pounds, or nearly three times the amount required.

The upward pressure of 30,000 pounds at the end of the beam must also be provided for.

For this we will rivet two $3'' \times 2\frac{1}{2}'' \times \frac{3}{8}''$ angle irons to the web, one on each side, the 3'' legs lying against the web.

The area of the cross section of these two angle irons is 4 square inches, which is abundant to take up a compression of 30,000 pounds.

Since the bearing capacity of a $\frac{3}{4}$ " rivet is 4,220 pounds, we will require $30,000 \div 4,220 = 8$ such rivets to transfer the strain from the angles to the web; 11 rivets are used, as shown in the figure.

It is important in such cases as this to secure a firm bearing of the lower ends of the vertical angle irons upon the flanges of the angle irons of the lower flange, in order that the former may take up the reaction, and prevent the crushing of the web plate.

Stiffening angles similar to those used at the ends should be riveted to the web at C and D.

Since riveted work is never perfect, and since, too, the strains in the flanges are along the lines of rivet holes, which do not coincide with the *center of forces* on the angles, it is plain that there must be some strain and distortion of the flanges, not provided for in the computation.

It is well, therefore, to use an abundance of rivets.

Taking the effective depth at 28 inches,

which it is customary to do, and which, we are assured, leads to "no essential error," we find the moment of resistance

$$=10,000 \times 7.5 \times 28 = 2,100,000.$$

Now $(2,100,000 - 1,883,000) \div 1,883,000 = .115$, which shows an error of $11\frac{1}{2}\%$.

The error committed by assuming the effective depth to be equal to that of the web plate is greater without than with flange plates; it increases as h increases compared with D .

It is usually between 10 and 20%, giving an average of perhaps 15%.

The distortion spoken of above is greater with than without cover plates; hence it is better to use cover plates sparingly, even at the sacrifice of some extra material at the ends of the angles.

It is generally better, too, to use one cover plate for each flange than two or more, even though the one requires more material than the two; for, in the first place, the heavy plate between two consecutive rivets will resist far more bending, as a column, than two plates of the

same combined thickness; and, again, many more rivets are required with two plates than with one.

We may now compute the weight of the beam.

We have—

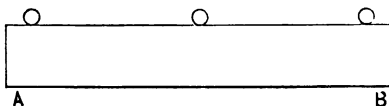
	Pounds.
1 web plate, $16' \times 28'' \times \frac{3}{8}'' \times \frac{1}{8}''$	560
4 angles, $4 \times 16' \times 10.4$	666
2 cover plates, $2 \times 10' \times 8'' \times \frac{5}{16}'' \times \frac{1}{8}''$	167
8 stiffening angles, $8 \times \frac{3}{4}'' \times 6.7$ sq. ins....	121
8 filling plates, $8 \times \frac{3}{4}'' \times 3'' \times \frac{1}{2}'' \times \frac{1}{8}''$	73
Rivet heads (say).....	33
	1,620

or 101 pounds per foot.

The weight of the beam exceeds the estimate by a few pounds only.

Should we dispense with the filling plates, which is often done, but not advisable, the weight of the beam would fall a little short of the estimate.

Fig. 60



The above design will apply equally

TABLE I.
MOMENTS OF INERTIA AND RESISTANCE.

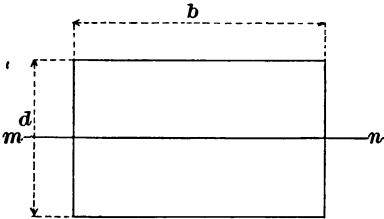
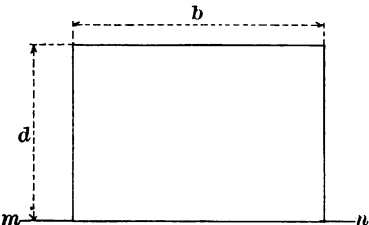
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 1	$\frac{1}{12}bd^3 = \frac{1}{12}Ad^2.$	$\frac{1}{6}bd^2 = \frac{1}{6}Ad.$
 Fig. 2	$\frac{1}{3}bd^3 = \frac{1}{3}Ad^2.$	$\frac{1}{3}bd^2 = \frac{1}{3}Ad.$

TABLE I. Continued.

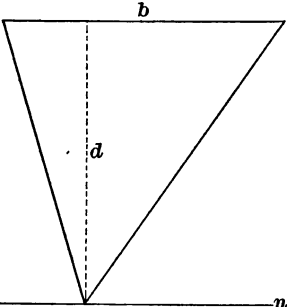
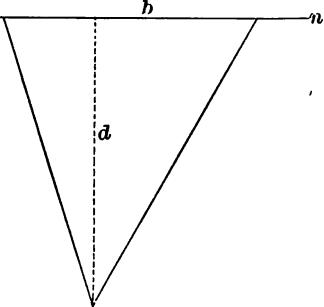
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 3	$\frac{1}{4}bd^3$	$\frac{1}{4}bd^2$
 Fig. 4	$\frac{1}{12}bd^3$	$\frac{1}{12}bd^2$

TABLE I.—Continued.

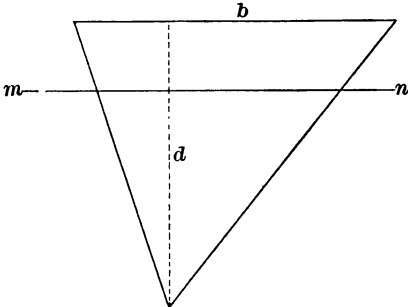
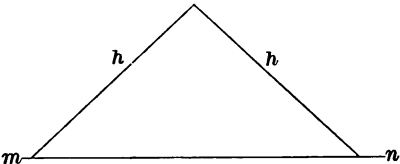
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 5	$\frac{1}{36}bd^3$	$\frac{1}{14}bd^2$
 Fig. 6	$.0417h^4$	$.059h^3$

TABLE I.—Continued.

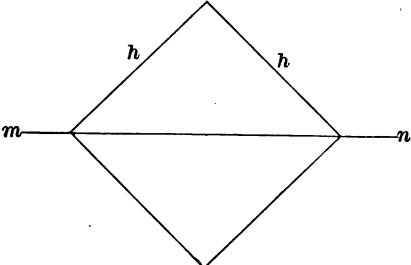
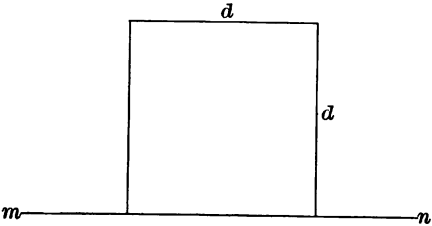
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 7	$.0834h^4$	$.1179h^3$
 Fig. 8	$\frac{1}{12}d^4$ $= \frac{1}{12}Ad^2$	$\frac{1}{6}d^3$ $= \frac{1}{6}Ad$

TABLE I.—Continued.

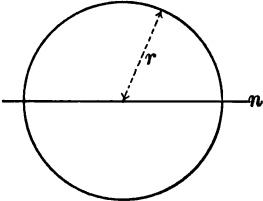
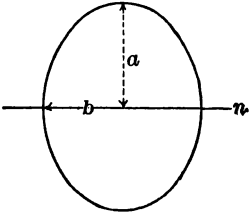
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{a}$
 Fig. 9	$\frac{1}{4}\pi r^4$	$\frac{1}{4}\pi r^3$
 Fig. 10	$\frac{1}{4}\pi a^3 b$	$\frac{1}{4}\pi a^2 b$

TABLE I.—Continued.

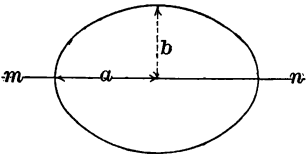
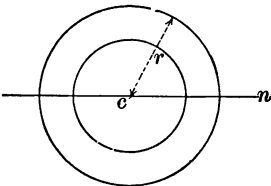
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{a}$
 Fig. 11	$\frac{1}{4}\pi b^3 a$	$\frac{1}{4}\pi b^2 a$
 Fig. 12	$\frac{1}{4}\pi(r^4 - r_1^4)$	$\frac{\frac{1}{4}\pi(r^4 - r_1^4)}{r}$

TABLE I.—Continued.

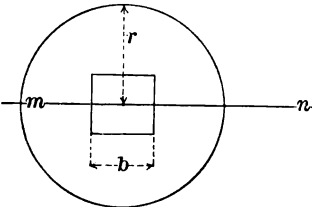
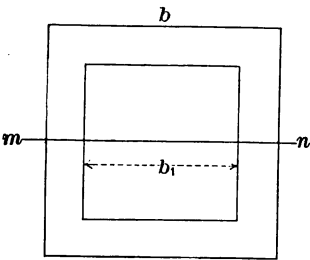
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{\bar{a}}$
	$\frac{1}{8}\pi r^4 - \frac{1}{8}b^4$	$\frac{\frac{1}{8}\pi r^4 - \frac{1}{8}b^4}{r}$
Fig. 13 	$\frac{1}{8}(b^4 - b_1^4)$	$\frac{\frac{1}{8}(b^4 - b_1^4)}{b}$
Fig. 14		

TABLE I.—Continued.

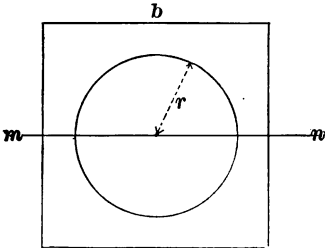
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{a}$
 The diagram shows a square with side length b. Inside the square is a circle with radius r. A horizontal line, labeled m at both ends, passes through the center of the circle. A dashed line connects the center of the circle to its top edge, labeled r.	$\frac{1}{18}b^4 - \frac{1}{4}\pi r^4$	$\frac{\frac{1}{18}b^4 - \frac{1}{4}\pi r^4}{b}$

Fig. 15

TABLE I.—Continued.

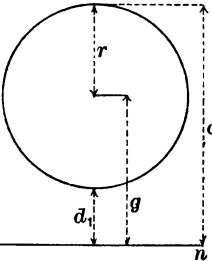
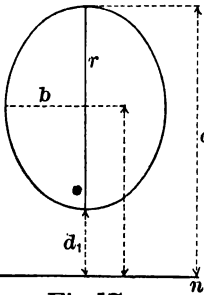
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 16	$a \left(d^3 - 2dr + \frac{5r^3}{4} \right)$	$a \left(d - 2r + \frac{5r^2}{4d} \right)$ $= a \left(d_1 + \frac{5r^2}{4d} \right)$
 Fig. 17	$a \left(d^3 - 2dr + \frac{5r^3}{4} \right)$	$a \left(d - 2r + \frac{5r^2}{4d} \right)$ $= a \left(d_1 + \frac{5r^2}{4d} \right)$

TABLE I.—Continued.

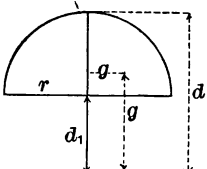
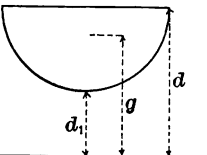
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{a}$
 Fig. 18	$a \left(d^3 - 1.15dr - \frac{2r^3}{5} \right)$	$a \left(d - 1.15r + \frac{2r^2}{5d} \right)$
 Fig. 19	$a \left(d^3 - .848dr + \frac{r^3}{4} \right)$	$a \left(d - .848r + \frac{r^2}{4d} \right)$

TABLE I.—Continued.

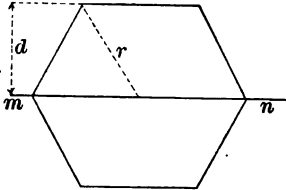
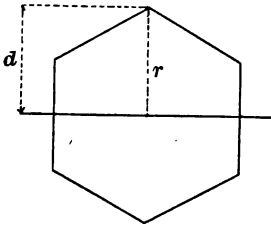
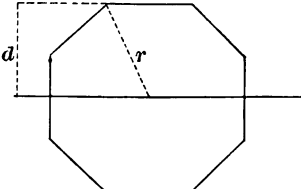
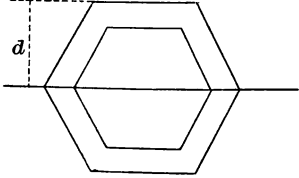
Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{d}$
 Fig. 20	$\frac{5}{8} d^4 \sqrt{8}$ $= 0.9623 d^4$ $= \frac{5}{8} r^4 \sqrt{8}$ $= 0.5418 r^4$	$0.9623 d^3$
 Fig. 21	$0.5418 d^4$ $= 0.5418 r^4$	$0.5418 d^3$

TABLE I.—Continued.

Form of Section.	Moment of Inertia I.	Moment of Resistance $\frac{I}{\bar{d}}$
	$\frac{1+2\sqrt{2}}{6}r^4$ $=0.6381r^4$ $=0.8761d^4$	$0.8761d^3$
Fig. 22		
	$0.8761(d^4-d_1^4)$	$\frac{0.8761(d^4-d_1^4)}{d}$
Fig. 23		

well to the track stringer, or to any track stringer or plate girder.

In case of the above track stringer, for example, the greatest reaction at each end is $\frac{24,000 \times}{2} = 36,000$ pounds.

The moment at the center of the stringer in inch-pounds is

$$36,000 \times 8 \times 12 - 24,000 \times 7 \times 12 = 1,440,000.$$

The moment of resistance of two 15" light Carnegie eye beams is, by Carnegie or our Table above,

$$2 \times 70.6 \times 10,000 = 1,412,000.$$

This shows that two eye beams slightly heavier than the above would be sufficient for track stringers for the assumed load.

WOODEN BEAMS.

The following Table gives the safe load, uniformly distributed, for rectangular oak or white or yellow pine beams, for a maximum fiber strain of 10,000 pounds per square inch. The table covers the

TABLE II.
SIZE OF BEAM.

Length in ft.	2 × 4	2 × 6	2 × 8	2 × 10	2 × 12	3 × 12	3 × 14	3 × 16
6	593	1333	2370	3704	5333	8000	10889	14222
7	508	1143	2032	3175	4571	6857	9833	12190
8	444	1000	1778	2778	4000	6000	8167	10667
9	395	889	1580	2469	3556	5333	7259	9481
10	356	800	1422	2222	3200	4800	6533	8533
11	323	727	1293	2020	2909	4364	5939	7758
12	296	667	1185	1852	2667	4000	5444	7111
13	274	615	1094	1709	2462	3692	5026	6564
14	254	571	1016	1587	2286	3429	4667	6095
15	237	533	948	1481	2133	3200	4356	5689
16	222	500	889	1389	2000	3000	4083	5333
17	209	471	837	1307	1882	2824	3843	5020
18	198	444	790	1235	1778	2667	3630	4741
19	187	421	749	1170	1684	2526	3439	4491
20	178	400	711	1111	1600	2400	3267	4267
21	169	381	677	1058	1524	2286	3111	4063
22	162	364	646	1010	1455	2182	2970	3879
23	155	348	618	966	1391	2087	2841	3710
24	148	333	593	926	1333	2000	2722	3556
25	142	320	569	889	1280	1920	2613	3413

sizes in general use. Observing, however, that the strength varies as the thickness and as the square of the depth, the strength of any size may be easily ob-

tained from some tabular size. Thus, a beam 2×16 will carry four times the load that one 2×8 will carry, or two-thirds the load that a beam 3×16 will carry.

Observing, too, that

$$9^3 = 8^3 + 4^3 \text{ nearly, } 10^3 = 8^3 + 6^3$$

$$18^3 = 16^3 + 8^3 \text{ nearly, and}$$

$$20^3 = 16^3 + 12^3, \text{ etc., etc.}$$

We see that a beam 9 inches in depth is but a trifle stronger than two beams of the same width, 8 and 4 inches in depth respectively. The same may be said of a beam 18 inches in depth compared with two beams 16 and 8 inches in depth. A beam 10 inches in depth has just the strength of two others of the same width, 8 and 6 inches in depth respectively; and the same is true of a beam 20 inches in depth compared with two others 16 and 12 inches in depth.

PLATE GIRDERS.

Table III. gives coefficients for determining the area required in flanges, allowing 10,000 pounds fiber strain per square inch of gross section.

TABLE

Distance between supports in ft.	Effective Depth of Girder in Inches.						
	12	14	16	18	20	22	24
10	.125	.107	.094	.083	.075	.068	.062
11	.137	.118	.103	.092	.082	.075	.069
12	.150	.129	.112	.100	.090	.082	.075
13	.162	.139	.122	.108	.097	.089	.081
14	.175	.150	.131	.117	.105	.095	.087
15	.187	.161	.141	.125	.112	.102	.094
16	.200	.171	.150	.133	.120	.109	.100
17	.212	.182	.159	.142	.127	.116	.106
18	.225	.193	.169	.150	.135	.123	.112
19	.237	.204	.178	.158	.142	.130	.119
20	.250	.214	.187	.167	.150	.136	.125
21	.262	.225	.197	.175	.157	.143	.131
22	.275	.236	.206	.183	.165	.150	.137
23	.287	.246	.216	.192	.172	.157	.144
24	.300	.257	.225	.200	.180	.164	.150
25	.312	.268	.234	.208	.187	.170	.156
26	.325	.279	.244	.217	.195	.177	.162
27	.337	.289	.253	.225	.202	.184	.169
28	.350	.300	.262	.233	.210	.191	.175
29	.362	.311	.272	.242	.217	.198	.181
30	.375	.321	.281	.250	.225	.205	.187

Distance between supports in ft.	Effective Depth of Girder in Inches.						
	26	28	30	32	34	36	38
10	.058	.054	.050	.047	.044	.042	.039
11	.063	.059	.055	.052	.049	.046	.043
12	.069	.064	.060	.056	.053	.050	.047
13	.075	.070	.065	.061	.058	.054	.051
14	.081	.075	.070	.066	.062	.058	.055
15	.087	.080	.075	.070	.066	.062	.059
16	.092	.086	.080	.075	.071	.067	.063
17	.098	.091	.085	.080	.075	.071	.067
18	.104	.096	.090	.084	.079	.075	.071
19	.110	.102	.095	.089	.084	.079	.075
20	.115	.107	.100	.094	.088	.083	.079
21	.121	.112	.105	.098	.093	.087	.083
22	.127	.118	.110	.103	.097	.092	.087
23	.133	.123	.115	.108	.101	.096	.091
24	.138	.129	.120	.112	.106	.100	.095
25	.144	.134	.125	.117	.110	.104	.099
26	.150	.139	.130	.122	.115	.108	.103
27	.156	.145	.135	.127	.119	.112	.107
28	.162	.150	.140	.131	.124	.117	.110
29	.167	.155	.145	.136	.128	.121	.114
30	.173	.161	.150	.141	.132	.125	.118
31	.179	.166	.155	.145	.137	.129	.122
32	.185	.171	.160	.150	.141	.133	.126
33	.190	.177	.165	.155	.146	.137	.130
34	.196	.182	.170	.159	.150	.142	.134
35	.202	.187	.175	.164	.154	.146	.138
36	.208	.193	.180	.169	.159	.150	.142
37	.213	.198	.185	.173	.163	.154	.146
38	.219	.204	.190	.178	.168	.158	.150
39	.225	.209	.195	.183	.172	.162	.154
40	.231	.214	.200	.188	.176	.167	.158

The Table gives, opposite the length and under the effective depth, the area required for a distributed load of 1,000 pounds. Hence, to find the area required for a given load, multiply the coefficient by the load in pounds and divide by 1,000.

For simplicity, the total depth may be taken for the effective depth; but this will of course give too large a co-efficient and too small an area. The error thus committed is approximately equal to the resistance of the web; and thus we see that, taking the total in place of the effective depth, virtually supposes the web effective in resisting longitudinal strains, though professedly ignoring it for such a purpose.

EXPLANATION OF TABLES IV., V. AND VI.

Rivets must be proportioned for transmitting the entire stress from one plate or group of plates, to another, taking into consideration the shearing and bending resistances, and the bearing surfaces. The requirements for bearing are fre-

TABLE IV.
Shearing, Bending Moment and Bearing Value of Rivets. Maximum fiber strain for bending
17,500 pounds per square inch.

Diameter of Rivet in Ins.	Area of Rivet.	Single Shear at 7500 lbs. per sq. inch.	Bending Moments in inch pounds.	Bearing value for different thicknesses of plate at 15,000 lbs. per sq. inch. (=Diameter of Rivet X thickness of plate X 15,000 lbs.)										
				$\frac{1}{4}$ "	$\frac{5}{16}$ "	$\frac{3}{8}$ "	$\frac{7}{16}$ "	$\frac{1}{2}$ "	$\frac{9}{16}$ "	$\frac{5}{8}$ "	$\frac{11}{16}$ "	$\frac{3}{4}$ "	$1\frac{1}{8}$ "	$1\frac{1}{2}$ "
$\frac{3}{8}$	.1104	828	91	1410										
$\frac{7}{16}$	.1503	1130	144	1640	2050	2810								
$\frac{1}{2}$	.1963	1470	215	1880	2340	3160	3690	5160						
$\frac{9}{16}$	.2485	1860	306	2110	2640	3520	4100							
$\frac{5}{8}$	.3068	2300	420	2340	2930	3870	4510	5630	6330					
$1\frac{1}{8}$	.3712	2780	558	2580	3220	4220	4920	5630	6360	7620				
$1\frac{1}{4}$	.4418	3310	725	2810	3520	4570	5330	6090	6860	8200				
$1\frac{3}{8}$	.5185	3890	921	3050	3810	4920	5740	6560	7380	8790	9670			
$1\frac{1}{2}$	.6013	4510	1151	3280	4100	4920	5740	6560	7380	8790	9670			
$1\frac{5}{8}$	.6903	5180	1415	3520	4390	5270	6150	7030	7910	8790	9670			
$1\frac{3}{4}$	.7854	5890	1718	3750	4690	5620	6560	7500	8440	9380	10310	11250		
$1\frac{7}{8}$	.8866	6650	2060	3980	4980	5980	6970	7970	8960	9960	10960	11950	12950	
$1\frac{1}{2}$	.9940	7460	2447	4220	5270	6380	7380	8440	9490	10550	11600	12660	13710	14770
$1\frac{5}{8}$	1.1075	8310	2877	4450	5570	6680	7790	8910	10020	11180	12250	13360	14470	15590

TABLE V.

Bearing Value of Pins for one Inch Thickness
of Plate.

Diameter of Pin. Inches.	Area of Pin. Sq. inches.	Bearing Value at 12500 lbs. per sq. inch. lbs.	Bearing Value at 15000 lbs. per sq. inch. lbs.
1	.785	12500	15000
1 $\frac{1}{8}$	.994	14100	16900
1 $\frac{1}{4}$	1.227	15600	18800
1 $\frac{3}{8}$	1.485	17200	20600
1 $\frac{1}{2}$	1.767	18800	22500
1 $\frac{5}{8}$	2.074	20300	24400
1 $\frac{3}{4}$	2.405	21900	26300
1 $\frac{7}{8}$	2.761	23400	28100
2	3.142	25000	30000
2 $\frac{1}{8}$	3.547	26600	31900
2 $\frac{1}{4}$	3.976	28100	33800
2 $\frac{3}{8}$	4.430	29700	35600
2 $\frac{1}{2}$	4.909	31300	37500
2 $\frac{5}{8}$	5.412	32800	39400
2 $\frac{3}{4}$	5.940	34400	41300
2 $\frac{7}{8}$	6.492	35900	43100
3	7.069	37500	45000
3 $\frac{1}{8}$	7.670	39100	46900
3 $\frac{1}{4}$	8.946	42200	50600
3 $\frac{3}{8}$	10.82	45300	54400
3 $\frac{1}{2}$	11.79	48400	58100

TABLE V.—Continued.

Diameter of Pin. Inches.	Area of Pin. Sq. inches.	Bearing Value at 12500 lbs. per sq. inch. lbs.	Bearing Value at 15000 lbs. per sq. inch. lbs.
$4\frac{1}{8}$	13.36	51600	61900
$4\frac{3}{8}$	15.03	54700	65600
$4\frac{1}{2}$	16.80	57800	69400
$4\frac{7}{8}$	18.67	60300	73100
$5\frac{1}{8}$	20.63	64100	76900
$5\frac{3}{8}$	22.69	67200	80600
$5\frac{1}{2}$	24.85	70300	84400
$5\frac{7}{8}$	27.11	73400	88100
$6\frac{1}{8}$	29.46	76600	91900
$6\frac{3}{8}$	31.92	79700	95600
$6\frac{1}{2}$	34.47	82800	99400
$6\frac{7}{8}$	37.12	85900	103100

quently neglected, notwithstanding such requirements, except for the smaller rivets, frequently determine the number of rivets to be used. The bending moments of rivets usually determine the number of the smaller sized rivets to be used. For example, the bearing value of a $\frac{3}{4}$ " rivet on a $\frac{3}{8}$ " plate is 4,220 pounds,

TABLE VI.

Maximum Bending Moments to be allowed on
Pins for Maximum Fiber Strains of 15000,
20000 and 22500 lbs. per square inch.

Diameter of Pin in Inches.	Moment for S=15000. Lbs. in.	Moment for S=20000. Lbs. in.	Moment for S=22500. Lbs. in.
1	1470	1960	2210
1 $\frac{1}{8}$	2100	2800	3140
1 $\frac{1}{4}$	2880	3830	4310
1 $\frac{3}{8}$	3830	5100	5740
1 $\frac{1}{2}$	4970	6630	7460
1 $\frac{5}{8}$	6320	8430	9480
1 $\frac{3}{4}$	7890	10500	11800
1 $\frac{7}{8}$	9710	12900	14600
2	11800	15700	17700
2 $\frac{1}{8}$	14100	18800	21200
2 $\frac{1}{4}$	16800	22400	25200
2 $\frac{3}{8}$	19700	26300	29600
2 $\frac{1}{2}$	23000	30700	34500
2 $\frac{5}{8}$	23600	35500	40000
2 $\frac{3}{4}$	30600	40800	45900
2 $\frac{7}{8}$	35000	46700	52500
3	39800	53000	59600
3 $\frac{1}{8}$	44900	59900	67400
3 $\frac{1}{4}$	50600	67400	75800
3 $\frac{3}{8}$	56600	75500	84900
3 $\frac{1}{2}$	63100	84200	94700
3 $\frac{5}{8}$	70100	93500	105200
3 $\frac{3}{4}$	77700	103500	116500
3 $\frac{7}{8}$	85700	114200	128500

TABLE VI.—Continued.

Diameter of Pin in Inches.	Moment for S=15000. Lbs. in.	Moment for S=20000. Lbs. in.	Moment for S=22500. Lbs. in.
4	94200	125700	141400
4 $\frac{1}{8}$	103400	137800	155000
4 $\frac{1}{4}$	113000	150700	169600
4 $\frac{3}{8}$	123300	164400	185000
4 $\frac{1}{2}$	134200	178900	201300
4 $\frac{5}{8}$	145700	194300	218500
4 $\frac{3}{4}$	157800	210400	236700
4 $\frac{7}{8}$	170600	227500	255900
5	184100	245400	276100
5 $\frac{1}{8}$	198200	264300	297300
5 $\frac{1}{4}$	213100	284100	319600
5 $\frac{3}{8}$	228700	304900	343000
5 $\frac{1}{2}$	245000	326700	367500
5 $\frac{5}{8}$	262100	349500	393100
5 $\frac{3}{4}$	280000	373300	419900
5 $\frac{7}{8}$	298600	398200	447900
6	318100	424100	477100
6 $\frac{1}{8}$	338400	451200	507600
6 $\frac{1}{4}$	359500	479400	539300
6 $\frac{3}{8}$	381500	508700	572300
6 $\frac{1}{2}$	404400	539200	606600
6 $\frac{5}{8}$	428200	570900	642300
6 $\frac{3}{4}$	452900	603900	679400
6 $\frac{7}{8}$	478500	638000	717800

while the double shear of the same is 6,620 pounds. Examples illustrating bending will be given subsequently. If in any given case the number of rivets required to furnish sufficient bearing, much exceeds the number for shearing or bending, it may be advisable to increase the thickness of the plate for the purpose of decreasing the number of rivets.

Pins must also be proportioned for shearing, bending and bearing; but one of the two latter determines, in almost every case, the size required.

Where groups of bars are connected to the same pin, as in the lower chords of truss bridges, the bars should be of a proper size, and be properly placed so as to make the bending strain, and consequently the size of the pin, as small as possible.

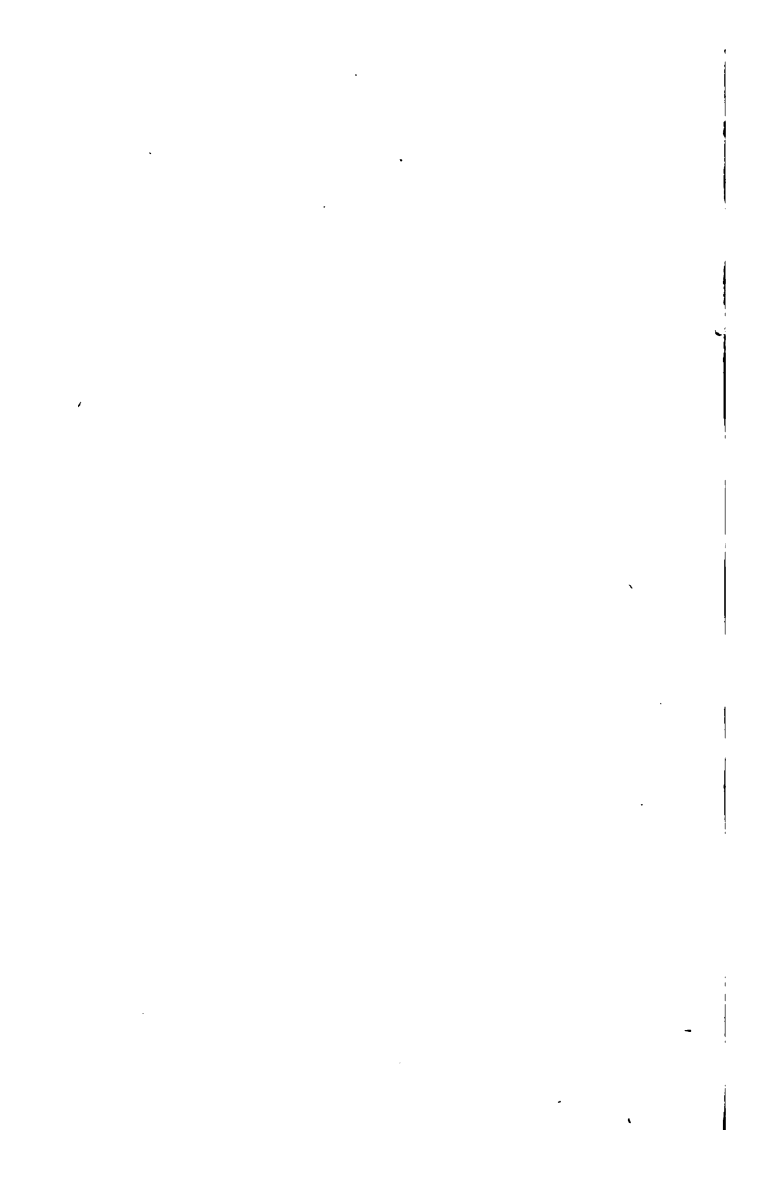
Examples.—Required the thickness of metal in the top chord of a bridge that will give sufficient bearing area for $3\frac{1}{8}$ " pin having to transmit a pressure of 60,000 pounds, the allowable pressure per

inch of diameter of pin for one inch thickness of plate being 12,500 pounds.

The bearing value of a $3\frac{1}{8}$ " pin for 1" thickness of plate = 39,100 pounds; therefore the thickness required = $\frac{39100}{12500} = 3\frac{1}{2}$ " nearly. Hence each of the two plates in the chord should be fully $\frac{3}{4}$ " thick.

The tension in the lower chord of a Pratt truss bridge, at the foot of the hip vertical, is 60,500 pounds. The chord on either side of the joint is formed of two bars $3\frac{1}{2}" \times \frac{7}{8}"$. what is the diameter of the required pin, allowing a fiber strain of 15,000 pounds?

The pull on each of a pair of bars (two bars "packed" close together in adjacent panels) is 30,250 pounds. The lever arm of this pull is the distance between centers of the bars = $\frac{7}{8}"$. Hence the moment is $30,250 \times \frac{7}{8} = 26,470$, which calls for a pin $2\frac{5}{8}"$ in diameter.



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